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ABSTRACT

Computer Algebra Systems (CAS) are computer systems for the exact solution of problems in symbolic form. The newspaper is designed to serve as a conduit for information and ideas on the use of CAS in education, especially in lower division college and university courses. Articles included are about CAS programs in several colleges, experiences using CAS in algebra and calculus classes, and using graphing calculators. (YP)

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Computer Algebra Systems in Education NEWSLETTER

Dept. of Mathematics, Colby College, Waterville, ME 04901
Number 1, January 1987

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[1] Introduction

The purpose of this newsletter is to serve as a conduit for information and ideas on the use of Computer Algebra Systems in education, especially in lower division courses. We will print ideas for classroom use, information on new products, and other material. If you wish to be on our mailing list for future issues, please fill out and return the form on the last page.

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[2] What are CAS?

Computer Algebra Systems (CAS) are computer systems for the exact solution of problems in symbolic form. This contrasts with the numerical analysis approach used in conventional computer languages such as FORTRAN or BASIC, where a numerical approximation is obtained. The ones of interest to us (MACSYMA, Maple, muMath, REDUCE, SMP) are interactive systems which allow the user to define an expression, apply an operation, and manipulate the output. For example, if the user wishes to expand

$$(a + b)^4$$

the user applies the expansion operator:

```
> expand((a+b)**4);
```

(where "**" indicates exponentiation and > the prompt from the CAS) obtaining as output the result, usually in two-dimensional format $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ rather than the one-dimensional format entered, (a + b)**4.

The standard operations include the use of the system as an unlimited precision desk calculator (do you want to know the factorial of 133?), algebraic simplification, calculus (differentiation, integration, power series), matrix algebra, systems of equations, differential equations; other operations often include number theoretic or special functions. There are also utility programs for expression manipulation, such as editing expressions and extracting parts of expressions. The systems may also allow numerical procedures such as numerical integration or graphing. If a procedure is not provided, these systems provide high-level programming capabilities to allow for user-written procedures as extensions to the system.

The initial CAS ran on mainframe computers and had a limited distribution. The most widely distributed system was REDUCE, which has been used in both Europe and the United States. Since the late 1970's some of the large systems (MACSYMA [pavelle] and REDUCE) were ported to minicomputers, a new large system (SMP [wolfram]) was developed for minicomputers, a medium sized system (Maple [char2]) was developed for minicomputers, and a small system (muMath [wilf]) was developed for microcomputers. A reference for general descriptions of CAS is [yun]. The descriptions of the systems as "large", "medium", and "small" refer to the amount of computer system resources required, the range of problems handled, and the complexity of problems handled.

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[3] CAS in Lower Division Education

Traditionally the focus of introductory mathematics courses has been on computation. The emphasis of classes, homework, and examinations has been on the student's ability to carry out certain algorithms: for differentiation, integration, limit, and series computations. With

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the widespread availability of Computer Algebra Systems which can carry out these algorithms better than most students, this emphasis has become anachronistic. The emphasis should be on concepts, understanding, and applications. The CAS can handle the routine computations while the student directs the analysis. This allows more examples and more complex examples to be considered. The student can focus on concepts rather than computational details. With CAS handling the tedious computations, the students can explore the concepts by looking at examples and looking for patterns. The CAS allow the integration of graphical, symbolic, and numeric methods in analyzing problems.

There have been efforts in the past to use computers as tools in teaching calculus. These efforts have focused on numerical methods and graphics, with both supplements and calculator/computer additions to standard texts. These efforts have not been successful in changing the way most calculus courses are taught. One reason is that most of these attempts at change require students to implement algorithms on computers (or at least on programmable calculators). The typical calculus course is already overloaded with material, and few instructors believe that the pedagogical advantages of implementing the numerical algorithms are worth the time (especially when the emphasis of the course remains on symbolic algorithms). Another factor is that much of calculus is concerned with symbolic operations, which are not suited to the floating point numerical methods. Stoutemyer [stoutemyer] points out that the arithmetic computations done with standard computer languages are not relevant to major portions of the mathematical curriculum (such as transformations of symbolic expressions, theorem proving, general solution of differential equations, and number theory) and even the numerical work encounters difficulties, since floating point arithmetic sometimes does not agree with the indefinite precision arithmetic done in school and available on computer algebra systems.

Computer algebra systems have a much greater opportunity to have an effect on instruction than the previous efforts for two reasons. First, these systems are easy to use. They require no programming effort by the student, and thus no class time for programming instruction. Second, the computer algebra systems perform the symbolic operations of calculus. Many of the CAS have graphics and numerical analysis routines built-in; if they aren't already built-in, then the desired algorithms can be added by the instructor or the CAS can be interfaced to existing routines. Thus CAS allow a unified approach to analysis, using symbolic, numerical, and graphical methods.

---

[4] Examples: Numerical Integration

In this section we present a sample section from a calculus supplement in which CAS, graphics, and numerical methods are used in numerical integration.

Prerequisites

(1) Local extrema of differentiable functions.

(2) Simpson's rule and its error estimate.

Discussion

It is usually impossible to obtain a useful antiderivative for $\int f(x)dx$ even when f is relatively simple. In these cases it is necessary to use approximations for the definite integral. Simpson's Rule,

$$\int_a^b f(x)dx = \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{2n-1}) + f(x_{2n})]$$

where

$$h = \frac{b-a}{2n} \text{ and } x_i = a + ih,\ i = 0, \ldots, 2n,$$

is one of the most commonly used elementary approximations.

The error between the approximation, A(n), and the integral is

$$|A(n) - \int_a^b f(x)dx| \le \frac{B(b-a)^5}{2880n^4},$$

where $|f^{(4)}(x)| \le B$ for all x in (a,b).

One reason for the popularity of Simpson's Rule is that the error term is inversely proportional to the fourth power of the number of intervals. Thus if the number of intervals is doubled, so that 2n is replaced by 4n, then the bound for the error is reduced by a factor of $2^4 = 16$.

In this section we will see that a variety of techniques, numerical, graphical and symbolic,

may be useful in applying Simpson's Rule to a problem. If the numerical Simpson's Rule or graphical routines are not provided by the CAS, they usually can be added using the programming language provided with the CAS.

Example

Suppose we want to compute $\int_1^4 \sin(x^2)dx$ with an accuracy of at least 10^{-4}.

The first thing we might try is to use our CAS to find an antiderivative:

```
> integrate(sin(x**2),x);
```

This will either fail, or, if our CAS is rather sophisticated, it may give the answer as an expression involving complex numbers and the error function, *erf*.

The next approach to the problem is to try a numerical approximation, in this case Simpson's Rule. First we need to determine the number of intervals, 2n, which is required to obtain the desired accuracy. That is, we want to find an n such that

$$\frac{B(b-a)^5}{2880n^4} \le 10^{-4}$$

for all x in [1,4].

To solve this inequality, we need to find B, a bound for $|f^{(4)}(x)|$ on [1,4]. We can compute $f^{(4)}$ using the symbolic capabilities of our CAS:

```
> d4f := diff(sin(x**2),x,4);
```

This gives:

$$d4f = 16\sin(x^2)x^4 - 48\cos(x^2)x^2 - 12\sin(x^2).$$

We need to estimate the maximum of d4f on [1,4]. We can try to estimate the maximum by using calculus. We know that since d4f is a differentiable function, its derivative,

$$d5f = 32\cos(x^2)x^5 + 160\sin(x^2)x^3 - 120\cos(x^2)x$$

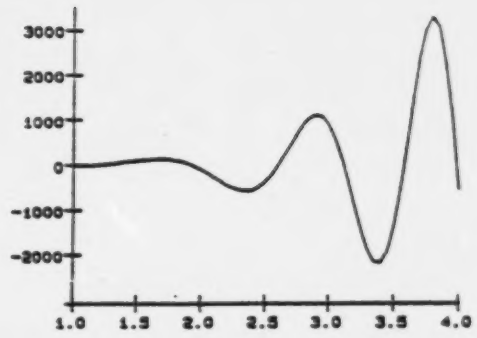
will vanish at a point in (1,4) where d4f has a maximum. If we ask our CAS to solve

```
> solve(d5f=0,x);
```

we may get several solutions or no solutions, since in a CAS, "solve" is designed to find exact, symbolic solutions rather than approximate numeric solutions. A good idea would be to look at the graph of d4f to see what to expect. We may be able to obtain a sufficiently good estimate of the bound B from the graph alone:

```
> graph(d4f,x,[1,4]);
```

gives



We see that d4f has several local extrema in (1,4), that $|f^{(4)}|$ attains its maximum on [1,4] at a point between x = 3.5 and x = 4, and that the maximum appears to be less than 3500. If we need a more precise answer, we can use the Bisection Algorithm to solve for a zero of d5f in [3.5,4] with an error of at most 10^{-6}:

```
> bisect(d5f,x,[3.5,4],10**(-6));
```

which returns 3.804649831. The value of d4f at this point is obtained by substituting the previous value returned (indicated by %) into the expression:

```
> substitute(x=%,d4f);
```

returns approximately 3382. Thus we have found a bound B for $|f^{(4)}|$ on [1,4] of 3500, or, more precisely, 3382.

We can now compute the number of intervals, 2n, which are required to guarantee the desired accuracy of 0.0001. We need to solve

$$\frac{3382(4-1)^5}{2880n^4} \le 10^{-4},$$

or

$$n^4 \ge \frac{10^4 * 3382 * 3^5}{2880}.$$

We can use the CAS as a calculator to evaluate

```
> evaluate((10**4*3382*3**5)/2880)**(1/4));
```

which returns approximately 41.1.

Since we need at most 2n subintervals, we will use 84.

We can now apply Simpson's Rule to the function

$$f(x)=\sin(x^2)$$

on [1,4]:

```
> simpson(sin(x**2),x,[1,4],84);
```

which returns the value 0.436870, or 0.4369 to four significant digits.

The number of subintervals used, 84, is based not on the specific function,

$$f(x) = \sin(x^2),$$

but on the bound for the fourth derivative. Eighty four subintervals are guaranteed to work for any function with the same bound. Thus the number of subintervals is likely to be conservative; fewer subintervals may give the desired accuracy. For this particular function, if we use 42 subintervals, then the value returned is 0.436943, which is 0.4369 using four significant digits.

Exercises

In the following exercises, give a complete report: problem, method of attack, and intermediate results.

(1) Find the integral of $f(x) = \frac{1}{1+x^3}$ on the interval [1, 2] with an error of at most 0.01.

(2) Find the integral of $f(x) = \sin(\sin(x))$ on the interval [0,1] with an error of at most 0.01.

(3) Find the integral of

$$f(x) = \sin(x^2 + 3x + 1)$$

on the interval [1, 2] with an error of at most 0.01.

(4) Find the integral of $f(x) = \frac{1}{\sqrt{1+x^3}}$ on the interval [1, 2] with an error of at most 0.01.

---

[5] Ask Cassie: Reader's Questions

Dear Cassie: Where can I find out about a computer algebra system for our computer? Reference [hosack] contains a comparison of several CAS with information on requirements and addresses of distributors.

Dear Cassie: Is there any instructional material available? Most of the work on computer supplements for lower division courses have concentrated on numerical and graphical applications. There is a great need for experimentation to develop applications which would use CAS or integrate the use of symbolic, numerical, and graphical methods. We would like to receive information on ideas and will publish them in future issues. We are especially interested in applications which lead students to explore and conjecture, rather than just compute.

---

[6] Further Information

For information or sample use with specific CAS, see the references [char2] (Maple), [hearn] (REDUCE), [jenks] (Scratchpad), [pavelle] (MACSYMA), [wilf] (MuMath), and [wolfram] (SMP). At the January San Antonio MAA meeting Brooks/Cole booth, there is a Maple demonstration and sample session handout available from the Maple developers at the Univ. of Waterloo.

For background and comparative information on CAS see [hosack] and [yun].

For information on the educational use of CAS see [char1], [freeze], [small], [stoutemyer], and [zorn].

---

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Computer Algebra Systems in Education NEWSLETTER*

Dept. of Mathematics, Colby College, Waterville, ME 04901
Number 2, September 1987

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[1] Who's Doing What and Where

St. Olaf College (Paul Zorn)

A three-year project funded by FIPSE to incorporate the use of CAS into elementary calculus. The goal of the project is to "help students understand calculus ideas more deeply and apply them more effectively." See the article, "CAS in Calculus at St. Olaf College," in Section [2].

Sloan Funded CAS Projects

Colby College (Don Small)

Develop a CAS Supplement for calculus courses. The Supplement will be independent of any particular calculus text and of any particular computer algebra system. The Supplement will be used this year in calculus courses at Colby College.

Develop a Computer Algebra Systems in Education *Newsletter* (the second issue of which you are now reading).

Denison University (Zaven Karian)

Design and teach an experimental calculus course which utilizes the capabilities of CAS. An expected outcome is a collection of exercises which through the use of CAS, will expose the underlying principles of calculus more convincingly than exercises found in current calculus texts.

Harvey Mudd College (Bob Borrelli)

Design and implement an interface between the Maple system and Math Lib, a scientific and mathematical software package developed at Harvey Mudd College. This enhanced package will then be used to provide a laboratory environment for calculus, linear algebra, and differential equations courses.

Oberlin College (George Andrews)

Create a set of computer laboratory exercises for the calculus. Each exercise is designed to fit into the traditional curriculum and provide a deeper understanding of some aspect of the calculus. Maple will be used as the laboratory software and students will write programs in the Maple language.

Rollins College (Doug Child)

Development of new methods for teaching problem-solving to calculus students utilizing CAS capabilities. The new methods will center around the development of computer languages based on mathematical terminology and problem-solving concepts and will lead to potentially useful tutoring systems. The Maple system will be used.

University of Waterloo (Peter Ponzo)

Generate a workbook of "Maple Problems" in calculus. These will be problems with multistep solutions which test a students ability to devise a solution procedure. To write a software interface which minimizes the need to learn a computer language (Maple). Also to write software which simplifies the writings of interactive tutorials in calculus. The Maple system will be used.

University of Saskatchewan (John Devitt)

Edit and distribute a Maple Newsletter.

University of Hawaii (David Stoutemer, Ralph Freese)

Develop a computer calculus class using muMATH. This past spring an experimental computer calculus II course using muMath was given as part of a regular calculus sequence.

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[2] Newsletters

CASE NEWSLETTER

The purpose of this newsletter is to serve as a conduit for information and ideas on the use of Computer Algebra Systems in education, especially in lower division courses. We will print ideas for classroom use, information on new products, and other material. If you wish to be on our mailing list for future issues, please fill out and return the form on the last page.

THE MAPLE NEWSLETTER

The purpose is to provide a forum for discussion of 1) the underlying philosophy for the basic design decisions of Maple; 2) new features, performance improvements, and mathematical extensions of the Maple system; 3) actual examples utilizing the Maple system; 4) new "packages" of routines and how they may be blended in with the Maple system.

Contributions and requests to be placed on the mailing list should be sent to J.S.Devitt, Department of Mathematics, University of Saskatchewan, Canada, S7N-OWO (306) 966-6114.

---

[3] "CAS in Calculus at St. Olaf College"
by Paul Zorn (St. Olaf College, Northfield, MN)

The St. Olaf mathematics department is finishing the first year of a three-year project to incorporate symbolic, numerical, and graphical computing into elementary calculus. Equipment for the project---a network of ten SUN-3 workstations running the computer algebra system SMP---was purchased through a grant from the College Science Instrumentation Program of NSF. (CSIP matches money raised from non-federal sources.) Course development is the main focus of the 3-year project. Financial support (for course materials, faculty released time, student helpers, etc.) comes from the Fund for Improvement of Postsecondary Education (FIPSE).

"Specifics:" Four SMP-aided sections of elementary calculus, enrolling about 120 students---1/4 of the total calculus enrollment---are taught each semester. Students in the project sections use SMP virtually every day to complete special homework sets. Problems are chosen to exploit SMP's various computational powers. (Producing such problems and other course materials---manuals, tests, etc.---is the largest daily task of project-supported faculty.) E.g., students might compare graphical, algebraic, and numerical methods of definite integration. Advanced student helpers staff an SMP laboratory, where the SUN machines are located, about 25 hours each week; they answer SMP-related, as well as mathematical, questions. The SUN machines are accessible from other terminals on campus; 15-20 simultaneous SMP jobs are possible.

SMP has been used, so far, mainly for homework. An overhead projection device produces acceptable (if imperfect!) classroom display of SMP graphical and alphanumeric output, but in practice, highly orchestrated use of SMP in lectures requires considerable planning and is subject to mechanical vagaries. More use of SMP in the classroom will probably come with experience. In the project's first semester, two of the four exams were in take-home format; for them, SMP was allowed.

"Philosophy:" The St. Olaf project does not primarily aim to computerize or automate elementary calculus. Although, arguably, many of the routine manipulations that---alas---fill calculus courses should be "mechanized," it would be worse than useless to do nothing else. The principal goals of the St. Olaf CAS project are to help students understand calculus ideas more deeply and apply them more effectively.

Such goals are admittedly uninteresting---who could disagree with them? The interesting question is how CAS's can help achieve them. We think that while a modicum of hand-computational facility is necessary, traditional courses overemphasize routine manipulations, and algebraic ones in particular. The result is a rigid, shallow, mechanical calculus, poorly suited to all but the most contrived applications. CAS's handle routine computations, with little overhead in distraction. In particular, they do the forbiddingly tedious and repetitive operations that graphical and numerical methods entail. Thus CAS's not only shift attention in a general way from mechanics to ideas, but also allow helpful and useful new calculus ideas and methods to emerge.

For more information: Department of Mathematics, St. Olaf College.

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[4] "Graphing Calculators as Symbolic Manipulators"
by Wade Ellis, Jr. (West Valley College, San Jose, CA)

The graphing calculators such as the Casio fx7000g and the Sharp 5200 allow the entry of functions in symbolic form. For example, the Graph key on the fx7000g allows the entry of a function as

$$y=\sin(x)+x^2.$$

The calculator can then use this symbolic function rule information to generate function value approximations numerically. Pushing this symbolic capability a little, numerical computations allows the calculator to add the two symbolic functions $f(x) = \sin(x)$ and $f(x) = x^2$ to form a third function, $h(x) = \sin(x) + x^2$, whose function values it can approximate. These function values are paired with domain values and these ordered pairs are graphed on coordinate axes (in an automatically or manually defined window). The resulting graphs, though often quite crude by computer graphics standards, are satisfactory for studying the behavior of functions. Approximate answers to the questions, "When is the function increasing/decreasing?" and "When is it bending up/down?" can often be determined from these graphs.

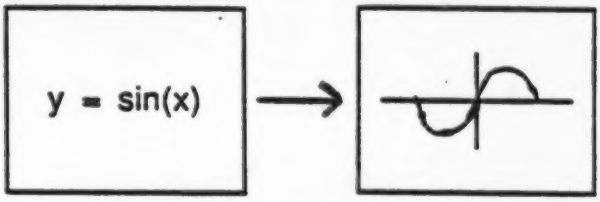
Indeed, this is a very low level of symbolic manipulation, but it is a very powerful capability that we should not overlook. In our desire to explore the full potential of computer algebra systems, we may be overlooking a gold mine of useful and accessible tools that our students can use effectively to solve problems. If we are interested in our students understanding function graphs and their usefulness, then perhaps we should pay closer attention to this symbolic capability.

For example, the standard question: Find the rational roots of the polynomial

$$f(x)=2x^3+x^2-8x-4.$$

can be answered very quickly using the graph of the function without using synthetic division and the rational roots theorem. If you, the instructor, claim that the graph will only give approximations, the student can check the result on the calculator to see if the function values are zero at 2, -2, and -0.5. If this is still not satisfactory, the graph will give the student excellent information on what values to try in the synthetic division process.

The calculators that students can now comfortably afford are powerful devices with some low level symbolic manipulation capability that will change how we approach some problems and some topics in the current curriculum. In some sense, they are very powerful symbol manipulators as the following diagram may indicate.



Here, the calculator "manipulates" a set of symbols representing a function into a symbol representing the graph of the function.

(For comments on the Hewlett-Packard HP-28C calculator, a more powerful symbol manipulator as well as a more expensive calculator than those referred to in the above article, see Tom Tucker's article "Calculators with a College Education?" in the January-February issue of FOCUS - ed.)

---

[5] Example: Discovery of Liebniz's Differentiation Rule
(submitted by John Hosack, Univ. of South Pacific, Fiji)

This is an exploratory exercise designed to lead students into discovering Leibniz's Rule for the nth derivative of the product of two functions. The procedure is to use a CAS to find the first few derivatives of the product (fg) and then to formulate a conjecture based on these derivatives. The final step is to verify the conjecture using an inductive proof. (The CAS commands illustrated are Maple instructions.)

Let f,g: R --> R be differentiable functions of order n. We denote the kth derivative by $f^{(k)}$. Thus

$$(fg)^{(0)} = fg \quad \text{and} \quad (fg)^{(1)} = f^{(1)}g + fg^{(1)}.$$

Question: What is the formula for $(fg)^{(n)}$?

We begin by defining two functions:

> f:=proc(x) defining expression for f end;

> g:=proc(x) defining expression for g end;

We now begin looking for a pattern by having our CAS compute $(fg)^{(k)}$ for the first few values of k. (Note: Double quote marks is the Maple instruction for the previous expression. Thus after having computed the first derivative with respect to x, diff(f(x)*g(x),x);, we instruct Maple to compute the second derivative by writing diff(,x).)

For k=1: > diff(f(x)*g(x),x);
yields:

$$diff(f(x),x)g(x) + f(x)diff(g(x),x)$$

or

$$f^{(1)}g + fg^{(1)}.$$

For k=2: > diff(,x);
yields:

$$diff(diff(f(x),x),x)g(x)$$

$$+ 2diff(f(x),x)diff(g(x),x)$$

$$+ f(x)diff(diff(g(x),x),x)$$

or

$$f^{(2)}g + 2f^{(1)}g^{(1)} + fg^{(2)}.$$

For k=3: > diff(,x);
yields:

$$diff(diff(diff(f(x),x),x),x)g(x)$$

$$+ 3diff(diff(f(x),x),x)diff(g(x),x)$$

$$+ 3diff(f(x),x)diff(diff(g(x),x),x)$$

$$+ diff(diff(diff(g(x),x),x),x)$$

or

$$f^{(3)} + 3f^{(2)}g^{(1)} + 3f^{(1)}g^{(2)} + g^{(3)}.$$

For k=4: > diff(,x);
yields:

$$diff(diff(diff(diff(f(x),x),x),x),x)$$

$$+ 4diff(diff(diff(f(x),x),x),x),diff(g(x),x)$$

$$+ 6diff(diff(f(x),x),x)diff(diff(g(x),x),x)$$

$$+ 4diff(f(x),x)diff(diff(diff(g(x),x),x),x)$$

$$+ diff(diff(diff(diff(g(x),x),x),x),x)$$

or

$$f^{(4)} + 4f^{(3)}g^{(1)} + 6f^{(2)}g^{(2)} + 4f^{(1)}g^{(3)} + g^{(4)}.$$

It appears that the pattern is that of the binomial expansion. (If this is not clear, compute some more derivatives.) Thus we have

Conjecture:

$$(fg)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}g^{(n-k)}$$

An induction proof can now be given establishing this conjecture as a theorem (Leibniz's Rule). The inductive step provides a nice application of the algorithm for obtaining the coefficients in the binomial expansion.

Example # 2: Prove (using mathematical induction)

$$\sum_{k=0}^{n} k^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12.}$$

(submitted by James White, Harvey Mudd College, Claremont, CA)

We begin by defining the function

$$sum(n) = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$$

and then verifying that $sum(1) = 1$.

> sum:= proc(n) $\frac{n^6}{6} + \frac{n^5}{2} + \frac{5*n^4}{12} - \frac{n^2}{12}$ end;

To verify that $sum(1) = 1$, we evaluate $sum(n)$ *at* $n=1$ by typing

> sum(1);

Our CAS returns the result 1.

We now check the inductive step, i.e show

$$(*) \quad sum(k) + (k+1)^5 = sum(k+1).$$

We ask our CAS to evaluate the left-hand side of (*)

> sum(k) + (k+1)^5;

The response is

$$\frac{k^6}{6} + \frac{3k^5}{2} + \frac{65k^4}{12} + 10k^3 + \frac{119k^2}{12} + 5k + 1$$

We now ask our CAS to evaluate the right-hand side of (*)

```
> sum(k+1);
```

The response is

$$\frac{k^6}{6} + \frac{3k^5}{2} + \frac{65k^4}{12} + 10k^3 + \frac{119k^2}{12} + 5k + 1$$

Thus our CAS has established the inductive step in the proof by showing that the left and right-hand sides of (*) are identical. This completes the proof.

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[6] News Items:

James White has developed and is marketing a new CAS for the IBM PC. His system, is called *CAL: The Mathematics Discovery Language.* It is published by Bluejay Lispware, P.O. Box 1708, Glendora, CA 91740 (818-335-6904).

The *Computer Algebra Supplement for Calculus* developed at Colby College is available to anyone who would like to use it in their class(es). The authors would appreciate receiving feedback as they plan to revise their Supplement next summer. Contact Don Small, Colby College, Waterville, Maine 04901.

CAS will be the topic of one of the Group Discussion Sessions at the National Research Council's national colloquium "Calculus For A New Century." The colloquium will be held in Washington, D.C. on October 28-29.

Warren Page is organizing a two and one-half hour session on CAS to be held during the January Math Meetings in Atlanta.

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[7]

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To receive subsequent copies of the newsletter, please complete, detach, and return this form to:

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Use of Computer Algebra Systems: current □, planned □ , uncertain □, unavailable □
Interest in contributing to the newsletter: yes □, no □, uncertain □
Suggestions for the Newsletter:

Computer Algebra Systems in Education NEWSLETTER*
Dept. of Mathematics, Colby College, Waterville, ME 04901
Number 3, January 1988

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[1] Events of Interest to CAS users.

The following events in 1988 may be of interest to those concerned with the use of computer algebra systems in education:

1. A course entitled "Computer Algebra" will be given by Prof. Wayne Patterson, Chair of the Department of Computer Science at the University of New Orleans. The course is scheduled on March 13-15 at the University of New Orleans and is cosponsored by the University of New Orleans and the Center for College/University Faculty Development, University of Georgia. The course will include a discussion of the principles of computer algebra systems as well an introduction to five of the better known systems. The registration fee is $175 and further information can be obtained from the Center for College/University Faculty Development, W. R. Zeitler, Director, 212 Aderhold Hall, University of Georgia, Athens, GA 30602 (Telephone 404-542-1763).
2. The use of computers in education will be an important part of the Sixth International Congress on Mathematics Education to be held in Budapest, July 27-August 3. Sessions of the Congress are divided into Action Groups, Theme Groups and Topic Groups. Action Group A5, Subgroup A5.3, is devoted to computing and mathematics at the college level. Meetings of A5.3 are planned to include both short paper presentations and group discussion. Some of the topics planned are:
   a.) educational technologies in general, including symbolic computing
   b.) effects of computing on the college mathematics curriculum
   c.) pedagogical and cognitive consequences of computing.

   For further information please contact one of the organizers:
   Paul Zorn, Department of Mathematics, Purdue University, W. Lafayette, IN 47907 (Telephone 317-494-1951)
   Warren Page, New York City Technical College, 300 Jay Street, Brooklyn NY 11202 (Telephone 718-643-3637).
3. A Computer Algebra Systems Workshop will be held at Colby College, July 21-23. The Workshop is for persons who have been active in the use of CAS in the classroom or involved in the development of CAS curriculum materials. The activities of the Workshop will center in three areas:
   a.) Sharing and evaluating of what has been done.
   b.) Discussion of the effects that CAS will have with respect to both course objectives and pedagogy (Calculus will be the primary course under discussion).
   c.) Development of future directions for utilizing CAS as an instructional tool.

   Persons interested in being invited to the workshop should send a description of their CAS activities to Don Small, Department of Mathematics, Colby College, Waterville, ME 04901 by February 8. Thirty participants will be selected and notified by February 15. Foundation support for travel, room and board for the participants is anticipated.

[2] NCTM Statement on Use of Computers

In September the Board of Directors of the National Council of Teachers of Mathematics adopted a statement outlining the official position of the NCTM on the use of computers in the learning and teaching of mathematics. While not mentioning CAS explicitly, the statement does appear to have implications for those interested in the use of CAS. The official NCTM position statement is:

Computer technology is changing the ways we use mathematics; consequently, the content of mathematics programs and the methods by which mathematics is taught are changing. Students must continue to study appropriate mathematics content, and they also must be able to recognize when and how to use computers effectively when doing mathematics. Teachers must know how and when to use the tools of computer technology to develop and expand their students' understanding of mathematics.

* Sponsored in part by a grant from the Sloan Foundation

It is the position of the National Council of Teachers of Mathematics that the computer is an appropriate tool that can be used in a variety of ways for the enhancement of mathematics learning, teaching, and evaluation. Changes are therefore needed in mathematics curricula, instructional methods, access to computer hardware and software, and teacher education:

- The content of school mathematics courses must be modified to reflect the changes brought about by computer technology. Curricular revisions should provide for the deletion of topics that are no longer useful, the addition of topics that have acquired new importance, and the retention of topics that remain important. In implementing revised curricula, educators must ensure that the time and emphasis allocated to topics are consistent with their importance in an age of increased access to technology. Instructional materials that capitalize on the power of computers must continue to be developed for students at all levels.
- Teachers should use computers as tools to assist students with the exploration and discovery of concepts, with the transition from concrete experiences to abstract mathematical ideas, with the practice of skills, and with the process of problem solving. In mathematics education computers must be instructional aids, not the object of instruction. Similarly, computer programming activities in mathematics classes should be used to support mathematics instruction; they should not be the focus of instruction. The amount of classroom time spent by mathematics students in learning a programming language must be consistent with the expected gains in mathematical understanding.
- Schools should be equipped with computers, peripherals, and courseware in sufficient quantity and quality for them to be used consistently in the teaching and learning of mathematics. Every classroom in which mathematics is taught should be equipped with computing hardware that includes a large-screen display device. Computer laboratories should be available to all students on a regular basis for the extended exploration of mathematical topics by individuals or groups. School systems must budget for the ongoing acquisition, maintenance, and upgrading of hardware and courseware for use in classrooms and computer laboratories at all grade levels
- All preservice and in-service teachers of mathematics should be educated on the use of computers in the teaching of mathematics and in examining curricula for technology-related modifications. Teachers should be prepared to design computer-integrated classroom and laboratory lessons that promote interaction among the students, the computers, and the teacher. Mathematics teachers should be able to select and use electronic courseware for a variety of activities such as simulation, generation and analysis of data, problem solving, graphical analysis, and practice.

Mathematics teachers should be able to appropriately use a variety of computer tools such as programming languages and spreadsheets in the mathematics classroom. For example, teachers should be able to identify topics for which expressing an algorithm as a computer program will deepen student insight, and they should be able to develop or modify programs to fit the needs of classes or individuals. Keeping pace with advances in technology will enable mathematics teachers to use the most efficient and effective tools available.
Changes in curricula and in the availability of hardware and courseware are not sufficient to guarantee that teachers use computers appropriately. Ongoing in-service programs must be readily available to help teachers take full advantage of the unique power of the computer as a tool for teaching and learning mathematics.

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[3] **CAS Project at Rollins College** by Doug Child (Rollins College, Winter Park, FL)

In 1985, Rollins became interested in using the computer algebra system Maple to teach calculus to entering, liberal arts students. The main problems perceived were that

a. Direct use of computer algebra systems would introduce a new, non-mathematical language into the mathematics classroom where each student already brings a non-mathematical language of his own.
b. Computer algebra systems were designed to assist people in doing mathematics; they were not designed as an aid for teaching mathematics. The author began experimenting with the way people interact with Maple, paying particular attention to the needs of Calculus I students. By Fall 1986, the first interface to Maple that would allow restricted, friendly use of Maple had been developed. It took advantage of Macintosh interface tools while conversing over a serial line with Maple running on a VAX. The interface and direct Maple were both used in Calculus I on an optional basis for the first time in Fall 1986. This experience reinforced the desire to continue development of the interface which, at that time, was useful for only a few special activities.

In May of 1987 the author visited the Maple Group at the University of Waterloo. Within one week, he and Benton Leong of the Maple group had combined efforts to produce the first stand-alone version of Maple for the Macintosh. It was constructed using Benton's version of Maple that ran on the Macintosh, but required the development system, MPW, and a new version of the author's Mac interface that was written using MacApp, a high level interface development package. Development of the new interface continued through the summer and by Fall a preliminary version of it was ready to be used in the author's experimental calculus class for math and science majors at Rollins. The experimental class met five days a week; one or two of these days were spent in a computer laboratory using the interface to learn how to solve certain classes of problems, to lay the foundation for understanding important concepts, and to develop skills important to the understanding of calculus. The experimental course effectively addressed many of the recomendations from the Tulane "Lean and Lively Calculus" Conference. Poor student algebra skills did not detract from the classroom experience and was not a factor in the course. Class time was spent discussing calculus concepts, teaching mathematical reasoning, learning the meaning of definitions and theorems, and discussing how to attack and rigorously solve problems while producing complete and clearly written solutions.

One of the main objectives of the interface is to provide a suitable environment for the teaching and learning of calculus. The work to this point has centered on developing various methods of creating and representing solutions to calculus problems. Maple, as well as all known computer algebra systems, were designed to be used as an assistant in the problem solving process. It responds with answers corresponding to certain kinds of requests. We have added the ability to create solutions that look similar to those found in most calculus texts. In addition, detailed information about how to solve many kinds of Calculus I problems has been provided.

Uses for the current environment include:

a. Checking homework problems, including in many cases most intermediate steps.
b. Teaching students how to think about certain types of problems, stressing the problem solving process (methods with justification).
c. Providing experimental introductions to the important concepts to improve students' understanding of these concepts.
d. Providing interactive text book discussions in which the student can examine different problems that are related to those presented.
e. Providing an alternate way of representing solutions that corresponds very closely to what instructors say as they describe how to solve problems. An interpreter is included that translates these descriptions into more traditional representations. Solutions can also be constructed by choosing menu items (minimal typing - more intuitive). In this case, the alternate form of the solution is created automatically.

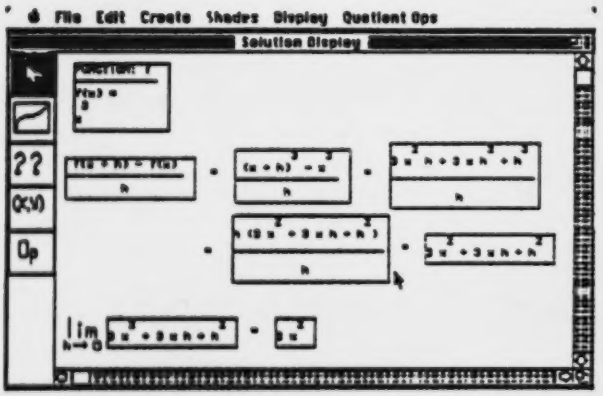
The teaching/learning environment has several modes of operation:

a. MOUSE DRIVEN MODE:

This method of operation follows the standard Macintosh interface guidelines. The Solution Display below was created by

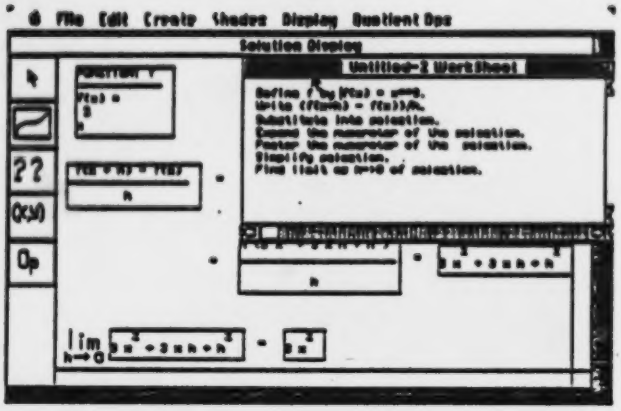
1. pulling down the Create Menu, selecting the Function Item, and typing '$f(x) = x**3$';
2. Pulling down the Create Menu, selecting the Expression Item, and the default expression $(f(x+h) - f(x))/h$;
3. The remaining argument was constructed by pulling down the right-most menu (Quotient Ops at the time the screen was printed) and selecting appropriate operations with the mouse.

The user can go back to any previous step and try a different operation if the expression is not simplifying as desired.



The result of this effort is a solution that looks like one from a calculus text or from the blackboard of a typical calculus classroom. Little typing is required so the student can focus on the process of solving the problem.

During experimention to obtain a solution, a 'script' is automatically created behind the Solution Display. For each create step or operation taken, a corresponding English-like instruction is created. These instructions are suggestive of the menu items selected, but the menu items are more brief. Once created, the script can be used to re-create the solution in the Solution Display window; simply point at the end of each line, click, and strike the 'Enter' key. The script can easily be edited in the usual Macintosh way (point, click, and type). The script in the WorkSheet window can be modified to solve a similar problem for a different definition of $f(x)$, for example, $f(x) = x * *6$. To do this, one need only change the '3' to a '6' and 'Enter' each line.



The script produced above is similar to what an instructor says in class as he writes the corresponding symbolic solution (like that in the Solution Display window) on the board.

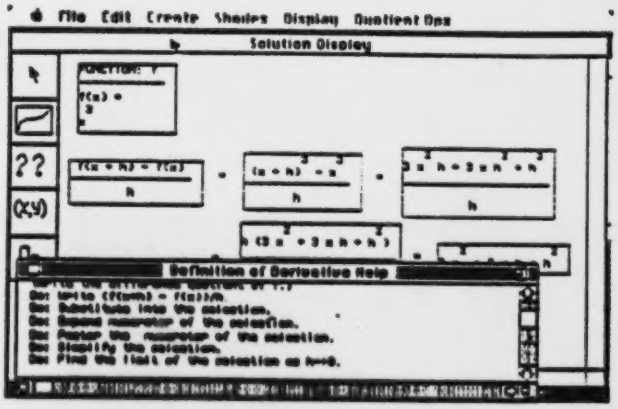
2. TEACHING MODE: Instructors can write scripts to address several pedagogical goals:

   a. To clearly tell students how to solve a certain type of problem by giving them a program or algorithm; see the example below.

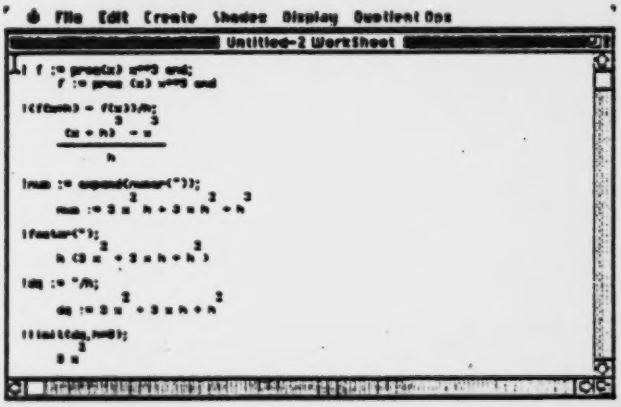
   b. To demonstrate important concepts like secant lines approaching tangent lines or slopes of secant lines approaching the slope of the tangent line.

   c. Describe a reasoning process that is helpful in solving certain kinds of problems.

Students can easily modify the scripts to solve related problems and can also write their own scripts to solve problems that are not solvable by any script given to them.



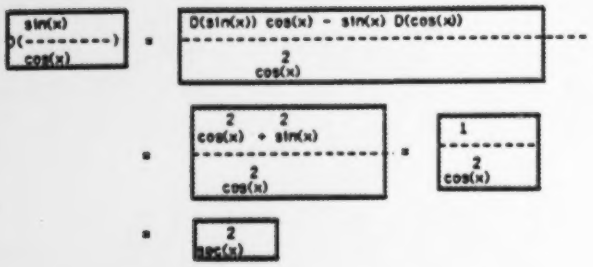
3. DIRECT MAPLE MODE: The dialogue below is a sample session in which we attempted to go through the same steps we used above (ones we would normally use to solve a 'definition of derivative' problem for $f(x) = x^3$). The appearance of the display is very different from that which appears in calculus books. It is also necessary to learn the (non-mathematical in character) commands and syntax of Maple. In this mode the student must type much more (everything after the '!' prompts) and must somehow remember a large collection of 'extra' language.



The following are examples of a few of the pedagogical uses of the interface.

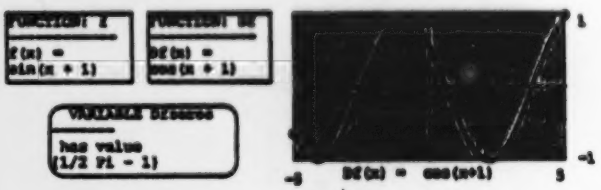
a. Creation of Equivalent Expressions

Reasoning chains can be entered, one link at a time. After the user has entered an expression which she thinks is equivalent to the current expression, it is checked for equivalence by Maple's *testeq* function. If *testeq* thinks the expressions are equivalent, the step is allowed. The user need only type the first block information, '1' in the fourth block and 'sec' in the fifth block. The rest is obtained by pointing and clicking. (See display below.)



b. Operations on Functions

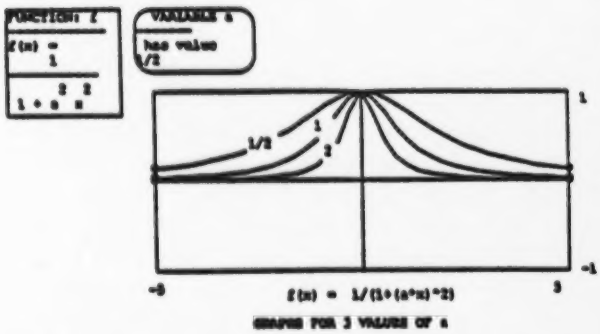
Operations that are commonly used in calculus are made readily available to the student. To obtain information for solutions to max/min problems and graphing problems, a student simply creates a function by entering something like $f(x) = \sin(x+1)$. She can then perform standard tasks like graphing, differentiating, finding zeros, and classifying f and its derivatives by pointing and clicking. An easy to use function calculator is available for each function known to the system. Manual bisection to find zeros can be achieved by entering the minimum amount of information possible. The only information typed to produce the display below was '$\sin(x+1)$'.



The function f is a COMPOSITION function.

c. Primitive Graphing Tool

Many graphing programs are available as separate programs, but this graphing facility is integrated into the environment and graphs can be placed anywhere in the Solution Display window. As a result, they can be used as part of the solution of a problem in a manner similar to that used with pencil and paper solutions. The graphing tool provided with the interface is fairly primitive and was designed to require the user to think about domain, range, asymptotes, etc. The tool relieves the user from having to evaluate the function and to plot points, but not from having to think.



d. 'Definition of Derivative' Problems

Students are placed in a closed system in which they can experiment with different operations on expressions in an attempt to perform the usual simplifications.

Help files are provided for many kinds of calculus problems. They are created by a typical editor so that they can be easily changed by both instructors and students. Help files contain sample scripts for solving problems. These help files attempt to capture the process that students must understand to solve the

problems. The script in PROBLEM 1 below captures the process for differentiating the composition of any two Basic Functions. The definitions of h, f and g need only be edited properly. Students may then write their own scripts to solve more difficult classes of problems. A partial help file will be found below the solution display.

Students traditionally have difficulty understanding compositions of functions and the chain rule. Use of this method has proven to be very successful. Students are provided (by the computer environment) with an endless supply of problems to work and they receive immediate feedback regarding the correctness of their efforts.

The following is an example of a help file which is available to the users via a pull-down menu item.

Help File for Solving Derivative Problems One solves derivative problems by recognizing functions as being constructed from certain fundamental building blocks that we will call Basic Functions. These Basic Functions all correspond to theorems that allow you to directly take a function's derivatives.

Examples of Basic Functions are

(1) polynomials like $3x^5 + 2x - 17$

(2) negative powers like x^{-5}

(3) root functions like $x^{1/3}$

(4) trig functions like $\sin x$,$\cos x$, $\tan x$, $\cot x$, $\sec x$, and $\csc x$.

THE METHOD:

For the function to be differentiated, (1) classify the function (2) write it in terms of Basic Functions, (3) differentiate each of the Basic Functions, (4) write the appropriate formula, and (5) substitute into that formula.

A DIFFERENTIATION PROBLEM THAT WILL BE USED TO DEMONSTRATE THE METHOD :

(1) Find $Dh(x)$ if $h(x) = \sin x^2$.

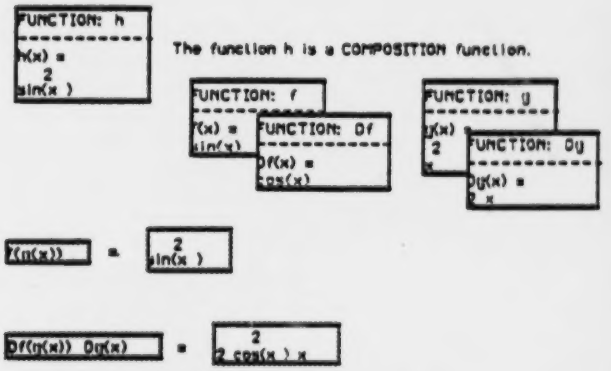
PROBLEM (1): Define h by $h(x) = \sin(x**2)$. Classify the function h. {Prepare to apply the Chain Rule.}

{Construct h from Basic Functions.} Define f by $f(x) = \sin(x)$. Define g by $g(x) = x**2$.

{Check construction(Optional).} Write $f(g(x))$. Substitute into selection.

{Apply the chain rule.} Write $Df(g(x)) * Dg(x)$. Substitute into selection.

The display below is the result of 'entering' the script under PROBLEM 1 above.



The experience developed in the experimental class aided in the design of the next version of the 'Calculus Teaching/Learning Environment' which will include the features of the current environment together with a hypertext help system, as well as features from spreadsheets, logic programming languages, and intelligent tutoring systems. It is hoped that a prototype will be ready for informal distibution sometime in the Summer of 1988. Anyone interested in this effort should contact me in the Mathematical Sciences Department at Rollins College, Winter Park, FL. 32789.

[4]

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Interest in contributing to the newsletter: yes ☐, no ☐, uncertain ☐
Suggestions for the Newsletter:

Computer Algebra Systems in Education NEWSLETTER*

Dept. of Mathematics, Colby College, Waterville, ME 04901
Number 4, June 1988

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[2] Learning by Teaching: A Novel Use for CAS in the Classroom.

[3] CAS Exercise with Variations

[4] Newsletter Request Form

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[1] Calculus Classroom Experiences with the HP-28C

John Kenelly
Clemson University

Even though the trial course was severely constrained by the requirement that we maintain a detailed department syllabus, the instruction of calculus was significantly improved by students having personal HP-28C units on loan from Hewlett Packard. The departmental syllabus specifies a daily schedule for topic coverage and homework problem to be assigned. To these constraints we added the assumption that some students in the experimental calculus I section would want to enroll in a subsequent regular section of calculus II (one did) and that the second semester experimental calculator section of calculus II would have "fresh" students from regular sections of calculus I added to its class roll (fifteen were). All the students were in the sections by choice and they were encouraged on the first day to change to a regular section, meeting at the same time, in case they had any misgivings about being in an experimental section or misgivings about the cash deposit requirement. Only one student in the total of seventy five students elected to transfer.

In calculus I, for the first time in my thirty years of teaching calculus, I was able to do a good job teaching the chain rule as a sequential step process. For three decades, I have explained to students that they should take the derivatives a step at a time and they should leave any "completed sections" alone. The A and B students historically mastered the point and the C students did not. Any chain rule test problem that involved three steps would consistently "sort" the lower ability students. In the past the C students could not apply the chain rule in a three stage problem. In the experimental section, we worked a

* Sponsored in part by a grant from the Sloan Foundation

in a three stage problem. In the experimental section, we worked a number of chain rule problems with the sequential differentiation calculator key. In this year's experimental section, on the examinations, a significant number of C students were able to work three and four step chain rule problems. I honestly feel that having the students "walk" the machine through the chain rule steps accomplished a simple instructional task that I had never successfully achieved in previous years.

On a much lower scale, I have also not been able to get my previous students to take any serious interest in why calculus derivatives must deal with variables expressed in radians (real numbers). All the talk and "test threats" simply would not spark their interest or concern. This year, I let them "accidentally discover" the calculator exhibit $\pi/180$ when a derivative of a circular function was taken with the unit in the degree mode. The class "would not shut up" until we had fully explained the result. I honestly feel that it was the threat of a machine "knowing" something that they did not understand that proved to be the stimulus to the students.

On the largest possible scale, the units completely turned around the way that the students looked at graphing and the calculus. The class started out with the graph and used the calculus to understand the process -- not the historically opposite direction. This was one of the most successful uses. In fact the students simply could not fathom that people had to use such long and involved calculations of derivatives just to find the general description of a curve. They looked upon the current methods (covered by necessity in the regular course syllabus) as antiquated.

The most telling experience in the calculus I section was the use of the HP-28C units on the tests. They were not available for the first hour test, and they were used on the second examination. As a simple item, I polled the class to see "if they wanted to use the units on the third test." I honestly feel that I would have had a riot on my hands had I denied the use of the calculators on the third test!

In the second semester, the students use the definite integrals to "check their answers" on a routine basis. They prefer the calculator check over the answers in the back of the book! I have asked them why, and they note that the book has only the even answers and the calculator will work them all. They also note that the calculator answer to a definite integral problem is a decimal number and immediately comparable to the calculated result, whereas the answer in the back of the book frequently contains varied expressions. But most of all, they noted that it was intellectually more satisfying to "work the problem two different ways." In these days of much maligned students, I found that response to be a surprise and satisfying. The students routinely work integrals numerically and this feature is the one on the machine that

gets the most use. This suggests that a numerical integral key should be on all the "low priced" graphing calculators like the Casio 7000, 8000 and the Sharp EL5200. This simple addition would make these units much more attractive to the calculus market. The absence of such a simple key on these units makes you wonder about the depth of market research that preceded their construction.

It goes without saying that the standard treatments of the numerical evaluations of integrals by right and left endpoint rectangles as well as the midpoint, trapezoid, and Simpson rules was completely changed by the availability of the HP-28C units. Once the students had written basic summation programs, they were able to very easily see evaluations improve by the use of different rules and decreasing mesh size. The time problems with the "slow" calculators gave them a special appreciation for the computing time price that you pay for increases in the number of subdivisions. I felt that the class had far more of a focus on the nature of these problems and almost no distractions from the messy arithmetic that usually accompanies these problems in a regular course. Since we had to "keep up" with the departmental syllabus, I did not try to estimate the magnitudes of the higher derivatives in order to evaluate error terms. I think that this might be more appropriate on a computer algebra system and a little bit beyond the best use of the hand held units. The students ran into memory problems when they tried to store several different integration programs and the 2K limitations in the HP-28C proved to be bothersome

The section has not gotten to series, but I expect that the HP-28C's will be a special help in the instructional process. I know that the small memory in the 28C units will prove to be a handicap and I wish that the class had the new HP-28S model. But, in reality, I really wish that Clemson University had a student computer algebra lab. I think that series instruction will be best approached in a combination of symbol manipulating calculators and computer algebra systems. The two will be perfect compliments to each other. Just as the pencil and the word processor form a happy pair -- the calculator and the micros will each support calculus instruction in their own special ways.

[2] Learning by Teaching: A Novel Use for Symblic Algebra in the Classroom

James E. White
Harvey Mudd College and
Jet Propulsion Labatories

One of the great challenges facing mathematics educators in the coming years will be the search for effective strategies for enriching their courses through the use of computers. While computers are finding their way into classrooms in increasing numbers as "smart blackboards" operated by instructors, their real promise lies

more in their potential to be individual student learning environments; they will be an interactive medium for mathematical exploration and discovery.

Because they deal with mathematical objects at a level of abstraction close to what we teach and encourage our students to use, Symbolic Algebra programs can be especially helpful in the creation of student learning environments. Such programs (when they have interactive graphics) can support genuine open-ended exploration in a wide range of topics from algebra through differential equations. The examples in this article are taken from a symbolic mathematics enviroment created by the author called CAL: The Math Processor. CAL operates on the IBM PC and compatible computers.

The teaching strategy that we describe in this note is the following.

Students may learn the basic steps of finding antiderivatives for functions by teaching CAL how to do it.

CAL contains a symbolic-algebra-based Expert System shell in which teachers or students may interactively teach it "rules" for the simplification and transformation of algebraic expressions. When a set of rules has reached a certain level of perfection, it may be saved and recalled later, or joined with other rules, for teaching, demonstration, or simply for calculation. These rules may be used to solve equations symbolically, calculate limits, apply identities (such as trigonometric or logarithmic identities), or, as we'll show below, to find antiderivatives for functions. This approach is largely inspired by the work of Alan Bundy [1]. Further discussion of CAL's structure as hierarchical microworlds of mathematical objects may be found in [2].

When CAL is given an expression to "simplify", it asks for the name of a variable. If asked to simplify something in the form:

Int(expression) (read this: Integral of expression)

CAL attempts to find an antiderivative for "expression" with respect to the variable supplied. All other variables are treated as constants, thus enabling CAL to calculate multiple integrals iteratively.

CAL does this by applying simplification rules. Initially, there are 6 such rules, very much like the ones we teach beginning students to enable them to calculate antiderivatives of polynomials. Starting with this set, we show how CAL can be taught (interactively and incrementally) additional rules to sharpen and extend its range.

Before listing the rules, we introduce some notation and terminology that can clarify how the Expert System works. If the discussion seems terse, please be patient. The examples that follow should clarify what is being said. Rules have three parts:

Here "query" is an algebraic expression containing special variables called "match variables". These variables are always preceded by a ?, e.g. ?a, ?b, etc. When given an algebraic expression "expression" to simplify, CAL looks through "expression" attempting to match "query" with any subexpression of "expression". This means that it looks for substitutions it can make for the match variables (multiple occurrences of the same variable must have the same substitutions) that render the query EQUAL to the subexpression.

If such substitutions can be found, they are called "bindings", and then they are substituted into "condition." The resulting condition, after substitution of the current bindings, is evaluated, and if the value is not false, then a replacement is made for the subexpression that matched the "query". If the value is false, then CAL looks for other matches for the query, and so on. Normally the "condition" is a proposition that evaluates either to T (meaning true) or NIL (meaning false).

The "replacement" is where the action happens. Here the bindings in force are substituted into the algebraic expression "replacement" and the result is inserted in the place of the matched subexpression of "expression". The rule is said to have "fired", and the "expression" is changed, hopefully for the better. CAL shows you the new expression, and asks if you want to continue. If you say yes, it goes back to the top of the rule list and tries to fire each one again in turn. It continues in this way until it can fire no more rules, then prints the answer, the simplified expression.

Following is the initial ruleset:

We mention 3 built-in propositions, and 1 built-in command that facilitate rule-writing. Below, VAR is the integration variable.

constant(a) is true if algebraic expression a does not depend on VAR

variable(a) is true if algebraic expression a does depend on VAR

indepvar(a) is true if algebraic expression a is, in fact, equal to VAR

The command: mksaf(expression) causes the expert system to generate a new variable and substitute it for expression. This new variable becomes the integration variable in this case.

1) int(?a + ?b) --> int(?a) + int(?b)

(This expresses the "additivity" of integration. It says that the integral of a sum is the sum of the integrals.)

2) int(?a) if constant(?a) -->?a * VAR

(This tells CAL how to integrate constants)

3) int(?a*?b) if constant(?a) -->
?a* int(?b)

3) int(?a*?b) if constant(?a) -->
?a* int(?b)

(This allows CAL to pull constants out from under the integral sign)

4) int(?a)if indepvar(?a) --> (?a^2)/2

(This tells CAL how to integrate the independent variable itself)

5) int(?a^?b) if and (indepvar(?a), constant (?b), not(?b = -1))
-->(?a^(?b+1))/(?b+1)

(This rule tells CAL how to integrate the variable raised to a constant power, different from -1.)

6) ?a*?var + ?b if and (constant(?a), constant(?b),indepvar(?var))
--
>mksaf(?a*var+?b)

(This is perhaps the most interesting rule. If CAL gets to this last rule, then it will replace any LINEAR expression in the integration variable (?var) with a NEW variable, multiplying the integral by the appropriate "scale" factor(1/?a). This is the rule for linear substitution.)

Now for some examples.

Example1: Find an antiderivative for

$$\left(x+\frac{1}{2}\right)^2$$

? int((x+1/2)^2)] ? x]

$INT(x^2) + INT(x) + INT(1/4)$
(Fires rule 1)

$INT(x^2) + INT(x) + x/4$ (Fires rule 2)

$INT(x^2) + x^2/2 + x/4$ (Fires rule 4)

$x^3/3 + x^2/2 + x/4$ (Fires rule 5)

Done! In general, CAL can integrate polynomials using this ruleset.

Example 2: Find an antiderivative for the square root of (1 - 2x).

? int(sqrt(1-2*x))] ? x]

SUBSTITUTING SUB1 FOR
- 2*x + 1
(Fires rule 6 and substitutes variable sub1 for 1 - 2*x)

$$\frac{INT((SUB1)^{1/2}}{-2}$$

$$\frac{-(SUB1)^{3/2}}{2*3/2}$$ (Fires rule 5)

$$\frac{-(-2*x+1)^{3/2}}{3}$$

Done!

Example3: Find an antiderivative for
$1/y + 3/y^3 - 5/y^4$

? int(1/y + 3/y^3 - 5/y^4)] ? y]

$INT(1/y) + INT(3/y^3) + INT(-5/y^4)$
(Fires rule 1)

(Fires rule 3)

Here it stops. It is unable to apply "constant power rule" 5. We ask to examine the PREFIX form of the answer.

```
 ? inspect answer]

 (+ (+ (INT (/ 1 y)) (* 3 (INT (^ (/ 1 y)
3)))) (INT (* -5 (^ (/ 1 y) 4))))
```

And now we see why. The independent variable y is not being raised to powers. Instead the quotient (1/y) (or in prefix form (/1 y)) is being raised to powers. This is an occasion to "teach" CAL a new rule, inserted between 5 & 6:

```
  int((1/?a)^?b)  if and (indepvar(?a),
constant(?b),not(?b = 1))
            ---> ?a ^ (1-?b) / (1-?b)
```

To give an idea what that is like, following is the precise input.

```
 ? rules]
  Rule number (Done to exit)
 ? 6]
  I]nsert, D]elete, or C]hange
 ? i]

  Expression to match
 ? int((1/?a)^?b)]
  Condition (t if none )
 ?
and(indepvar(?a),constant(?b),not(?b =
1))]
  Substitution
 ? ?a^(1-?b)/(1-?b)]
```

Now with that rule inserted, we ask CAL to try Example 3 again.

```
 ? int(1/y + 3/y^3 - 5/y^4)] ? y]
```

$$INT(1 / y) + INT(3 / y^3) + INT(-5 / y^4)$$

(Fires rule 1)

$$INT(1 / y) + 3 * INT(1 / y^3) - 5 * INT(1 / y^4)$$

(Fires rule 3)

$$INT(1 / y) + 3 * INT(1 / y^3) - 5 * INT(1 / y^4)$$

(Fires new rule 6 - no visible effect, but the internal form is changed)

$$INT(1 / y) + \frac{-3}{2 * y^2} + \frac{5}{3 * y^3}$$

(Fires rule 5

Notice, however, that CAL was unable to integrate 1/y. Neither can most beginning students. When she learns how, a student could teach CAL a new rule

```
int(1/?a)  if indepvar(?a) --->  ln(?a)
```

Example 4: Find an antiderivative for x/y with respect to y)

```
 ? int(x/y)]
 ? y]

   x*INT(1/y)

   x*ln(y)
```

Done! So now CAL is a bit smarter.

Now this enhanced ruleset can be enriched with further exponential and logarithmic identities, rules for integrating trigonometric functions, and so on. As new facts are learned, the student can enlarge CAL's rule-set and experiment with the results. This can lead to the creation by students of unique and separate environments, each of which displays a different "competence" based on what it was "taught".

References:

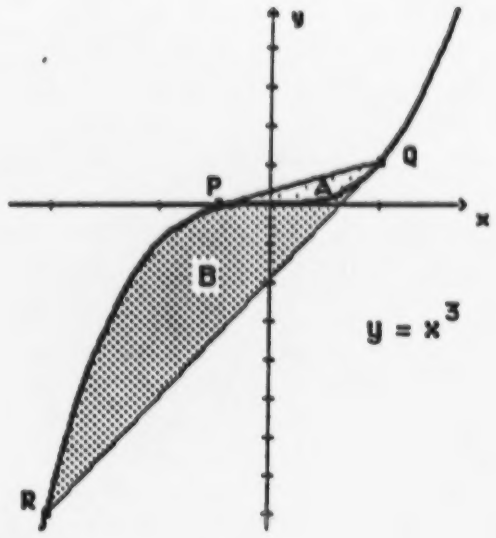
1] Bundy, Alan, The Computer Modelling of Mathematical Reasoning, Academic Press, 1983

2] White, James, "Teaching with CAL: A Mathematics Teaching and Learning Environment for Microcomputers", College Mathematics Journal (to appear)

[3] CAS Exercise with Variations (Problem submitted by Ian McGee, Univ. of Waterloo)

Cubic Curve Property: Suppose P is any point on the graph of $y=x^3$. The tangent at P crosses the curve at Q, and A is the area between the curve and the line seqment PQ. Similarly, the tangent at Q meets the curve again at R, and B is the area between the curve and QR.

Show that B is always 16 times as great as A for every choice of the point P.




```
# Define the curve and its derivative:
>y:=x^3;
```

$$y := x^3$$

```
>yp:=diff(y,x);
```

$$yp := 3\,x^2$$

```
# Assume the coordinates of the point P are: (k,k^3), say. Then the tangent line PQ has equation:
>PQ:=subs(x=k,y) + subs(x=k,yp)*(x-k);
```

$$PQ := k^3 + 3\,k^2\,(x - k)$$

```
# Determine the x-coordinate of the intersection of PQ and the cubic:
>rts:=solve(PQ=y,x);
```

$$rts := -2\,k,\ k,\ k$$

```
>xQ:=rts[1];
```

$$xQ := -2\,k$$

```
>if rts[1]=rts[2] then xQ:=rts[3] fi;
#The double root "k" is the point of tangency. The other point is the x-coordinate of Q.
# Now calculate the area A:
>areaA:=int(PQ-y,x=k..xQ);
```

$$areaA := 27/4\ k^4$$

```
#write the equation of the tangent line QR:
```

```
>QR:=subs(x=xQ,y) + subs(x=xQ,yp)*(x-
xQ);
                QR := - 8 k^3 + 12 k^2 (x + 2 k)
# Find the x-coordinate of R:
>roots:=solve(y=QR,x);
                    roots := 4k,  - 2 k,  - 2 k
>xR:=roots[1];
                   xR := 4 k
>if roots[1]=roots[2] then xR:=roots[3]
fi;
#Now calculate the area B:
>areaB:= int(y-QR,x=xR..xQ);
                   areaB := 108 k^4
# Compare areas:
>ratio:= normal( areaB/areaA );
                   ratio := 16
>done
```

Variation 1: The original problem shows that the property: area B = 16 area A is true for any point P on the curve $y = x^3$. Show that this property also holds for the curve $y = x^{1/3}$.

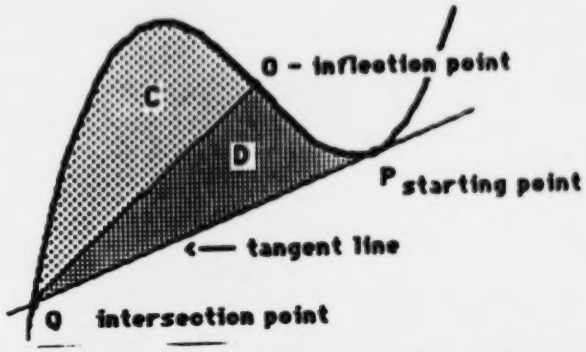
Variation 2: Show that the property area B = 16 area A holds for *every* curve of third degree. Use $y = ax^3 + bx^2 + cx + d$ and use (x_1, y_1) for the coordinates of the point P.

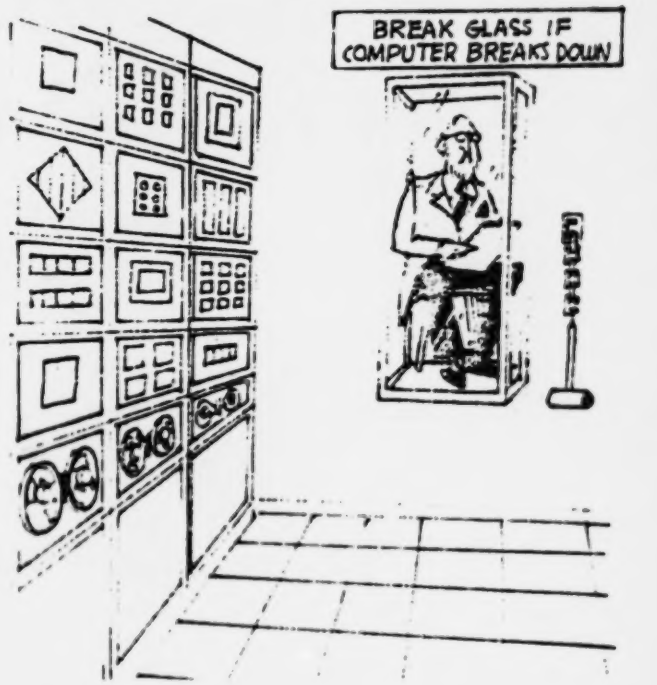
Variation 3: Perform the same sequence of operations as in the original problem, but on the function $y = ax^{1/3} + bx$. Compute the ratio of area B to area A.

Variation 4: Suppose P is any point on the graph of the polynomial

$$f(x) = ax^3 + bx^2 + cx + d,$$

a tangent line is drawn to intersect the cubic again at Q say. (Note that points P and Q will always be on opposite sides of the inflection point)). Show that the area between the curve and the line PQ is divided in the ratio 16:11 by the line OQ.





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Use of Computer Algebra Systems: current , planned ,
uncertain , unavailable

Interest in contributing to the Newsletter: yes , no , uncertain

Suggestions for the Newsletter:

Computer Algebra Systems in Education **NEWSLETTER***

Dept. of Mathematics, Colby College, Waterville, ME 04901
Number 5, January 1989

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Getting Started with a Computer Algebra System

Zaven Karian - Andrew Sterrett
Denison University

With support from the Sloan Foundation, in the summer of 1987 Denison University began a pilot program to incorporate symbolic computation into its calculus curriculum. After reviewing the relative merits of various computer algebra systems (CAS's) available, we chose Maple. A grant from the National Science Foundation permitted us to purchase equipment that would be controlled by the department and dedicated to the project.

We chose the Sun Microsystem 3/280 computer, now upgraded to the SUN 4/280 with 32M bytes of memory. One terminal, with an attached projector, was installed in the classroom where most of the calculus instruction takes place, All faculty offices are connected to the SUN system and we have a laboratory with 18 terminals available to students.

During the fall of 1987 the authors each taught one section of Calculus I using CAS. In the spring semester we taught one section of Calculus II and repeated Calculus I. Our limited experience does not transform us into experts in the educational uses of CAS, but we have learned some things that don't work as well as some that do. Consequently, with some misgivings, we offer the following suggestions to be considered by those contemplating a similar effort.

DON'T try to incorporate CAS into your calculus program by yourself. A tremendous amount of planning is required to introduce CAS into the curriculum. Decisions with regard to the particular CAS to be used and the hardware on which it will be run, revisions of course syllabi, and the development of new classroom demonstrations and assignments are more likely to be sound ones if this is a cooperative, possibly a departmental, venture.

* Sponsored in part by a grant from the Sloan Foundation

DO provide an overhead projector in the classroom so that the usefulness of CAS in solving problems can easily be demonstrated.

DON'T assume that CAS will free the student who has a poor background in algebra to concentrate on setting up problems. These students now must master the syntax of a new language as well as set up the problem.

DON'T expect students to become self-starters in the use of CAS. We believed that Maple would be so user friendly that very little instruction would be required in class, and we continue to be puzzled by the apparent difficulty that many students seem to have with the syntax.

DO involve as many instructors/sections in an experiment as possible. For a first effort, a less-ambitious experiment, where all instructors use CAS in class to demonstrate its potential, is more likely to be satisfactory than an experiment where one or two sections are expected to master CAS in addition to calculus.

DO understand that doing one's assignment in a laboratory may inconvenience some students.

DO expect to spend more than the usual amount of time preparing for and teaching a course that involves CAS. Regardless of the text, you are not likely to be satisfied with the problem sets presently available, and countless hours can be spent trying to find appropriate exercises.

DO expect to omit a topic or two from your syllabus. Instruction on how to use a CAS system and classroom demonstrations take time, and something has to give.

DO find a way to make the use of CAS an integral part of your course. We did not accomplish this to our satisfaction during our first year because the assignments that required CAS were not tied closely enough to current concepts and were not evaluated sufficiently often.

In order to integrate CAS as an effective tool to solve calculus problems, our department decided to alter the format of the first-year calculus sequence in 1988-89. All sections of Calculus I and II will consist of three lecture/recitation classes and a hands-on laboratory instead of our customary four lecture/recitation classes. Laboratory assignments will become a significant part of all courses and will be intended to reinforce understanding of concepts currently being developed.

DO try to inhibit your natural tendencies to "cover" the current, massive calculus texts and take time in class to demonstrate how much CAS contributes to <u>your</u> ability to solve problems quickly and accurately.

DON'T conclude that CAS, by itself, will promote mathematical thinking. There will be occasions, however, when CAS can be used to look for patterns and generalizations. For example, students can be asked to find expressions for the first, second, third, and fourth derivatives of f(x)*g(x), enough derivatives to speculate on a general formula for the nth derivative.

DO be prepared for surprises. We gave our students a definite integral to evaluate, one for which an antiderivative does not exist, intending that they would use a Riemann sum to approximate the integral. Instead, they entered the following Maple command:

```
> int(sqrt(1 + cos(x)^2), x = 0..Pi);
```

When students learned that Maple would not produce a numerical result, rather than proceed to evaluate a Riemann sum as we had intended, they simply entered:

```
> evalf(");
```

which converts expressions to floating point numbers. We had not realized that our CAS had a built-in subroutine that provides approximations for definite integrals when antiderivatives cannot be found, but the word quickly got around our small campus that Maple has this capability.

We hope that our suggestions do not imply that we are discouraged by our first attempts to improve our students' understanding of calculus by making CAS available to them. On the contrary, we are encouraged by the potential that computer algebra systems have to help mathematics students and we look forward to refining our methods in 1988-89.

[2] CAS Regional Workshops

The National Science Foundation has funded a proposal to conduct ten 2-Day Regional Workshops and five 5-Day Regional Workshops over the next two years. The grant covers room and board expenses for the participants.

The 2-Day Workshops are directed towards persons who are interested in "getting started" using a CAS. The Workshop program will contain approximately eight hours of lecture-discussion and eight hours of "hands-on" work. Participants will gain an awareness of CASs and their possible roles in improving calculus instruction and will receive enough background to allow further independent work. Participants will leave the Workshop with the beginnings of a specific CAS curriculum innovation, such as a seminar or class project, that can be carried out in the participants home institution.

The 5-Day Workshops are directed towards persons who already have experience using a CAS in their teaching. Participants will receive a "total immersion" experience in the use and development of CASs as teaching/learning tools through lectures, small group projects, demonstrations, individual assignments, discussions. Participants will be involved in (calculus) curriculum development and will receive sufficient background information to make informed judgements on CAS related curriculum issues.

The Program is administered by the Project Director Don Small (Colby College) and an Advisory Committee consisting of: Doug Child (Rollins College), Stan Devitt (Univ. of Saskatchewan), Wade Ellis, Jr. (West Valley Community College), Zaven Karian (Denison Univ.), Arnie Ostebee (St. Olaf College, Warren Page (New York City Technical College), and Paul Zorn (St. Olaf

College). The following Workshops are presently scheduled:

March 17-19: Pomona College (CA)
Don Small, Instructor
Joanne Badagliacco, Contact Person

March 17-18: West Valley Community College
Wade Ellis, Jr., Instructor

April 14-16: Rollins College (FL)
Doug Child, Instructor
Linda Gentry, Contact Person

May 13-15: Trinity Univ. (TX)
Stan Devitt, Instructor
Don Bailey, Contact Person

July 24-28: St. Olaf College (MN) (5-Day)
Paul Zorn, Instructor
Mike Henley, Instructor
Arnie Ostebee, Contact Person

Further information concerning the program can be obtained from Don Small or one of the contact persons.

[3] Subcommittee on Symbolic Computation

Prof. Lynn Steen, Chair of the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America (MAA), announced the formation of a Subcommittee on Symbolic Computation (SSC). The general charge to the Subcommittee is to coordinate the existing disparate activities relating to CAS, to promote new initiatives, and to disseminate the work being done in this area to the larger mathematics and science community. The Subcommittee's activities will be supported for the first two years by a grant from the Sloan Foundation.

The Subcommittee is chaired by Zaven Karian (Denison Univ.). Other members are: James Daniel (Univ. of Texas at Austin), Wade Ellis, Jr. (West Valley Community College), Joan Hundhausen (Colorado School of Mines), Robert Lopez (Rose-Hulman Institute of Technology), Arnold Ostebee (St. Olaf College), Warren Page (New York City Technical College), Don Small (Colby College), David Smith (Duke Univ.), Jerry Uhl, Jr. (Univ. of Illinois). The first meeting will be held in Phoenix, AZ during the MAA Meetings.

[4] A New CAS: Derive

John Hosack
Univ. of South Pacific

David Stoutemyer, the father of muMATH, has introduced a new Computer Algebra System, Derive. Like muMATH, it is an IBM-PC compatible program requiring only a minimal configuration (no hard disk).While other Computer Algebra Systems are more powerful than muMATH or Derive, they do not run on IBM-PC compatible machines, which are by far the most common machines used by students. Thus a new IBM-PC compatible CAS is an important event for those interested in the instructional uses of Computer Algebra Systems.

The major advances of Derive over muMATH are graphics and an improved user interface. Derive does two and

three dimensional graphics on color or monochrome graphics monitors. The user interface is highly graphical and menu-driven. (A sample display screen is pictured in an advertisement in the December 1988 Notices of the American Mathematical Society on page 1667.) The user enters expressions on a command line. These are then displayed in two-dimensional format. The user can apply a large collection of built-in operations to these expressions. Expressions or parts of displayed expressions can be selected by means of the arrow keys for use in building new expressions. The system also provides on-line help.

The built-in functions include many from real and complex arithmetic and calculus, some from matrix algebra, and a few from statistics. The mathematical capabilities include numerical integration when the symbolic integrator fails. There is no programming language to allow users to develop their own functions, although expressions can be saved from and read into the system.

I found Derive easy to use and surprisingly powerful for its size. Further information can be obtained from:
The Soft Warehouse,
3615 Harding Avenue, Suite 505,
Honolulu, HI 96816
(telephone (808) 734-5801).

[5] Script for a CAS Classroom Exercise

Don Small
Colby College

This exercise requires a classroom with a chalkboard and a CAS connected to a projection device. The exercise is to obtain a global sketch of

$$f(x) = \frac{x^5 - x^4}{x^5 - 6x^4 + 5x^3 + 26x^2 - 48x + 18}$$

The objectives of this exercise are to illustrate use of the different environments of a CAS coordinated with the chalkboard and to involve students in a problem solving experience. "(Q)" is used to suggest the instructor probe the students' understanding of what is happening at that point. The chalkboard is used primarily as a reference board. Thus one begins by writing the above equation on the board and drawing a pair of axes. As information is obtained, it will be indicated on the axes. The above equation is entered into the computer prior to class.

What is the first step (Q)? Students will probably suggest using the CAS plot command to obtain a graph. When this is tried, the result does not give enough information due to the scaling involved (Q). (Why is scaling a problem when plotting the graph of a rational function?) E.g., a plot over the domains [-10,10] and [-5,5] indicate a vertical asymptote near x=3, but no others. Adjusting the scaling may provide more information, but is not sufficient for a global sketch. Since the degree of the

denominator is odd, it is possible that there is only one asymptote (Q). More information is needed.

How are the vertical asymptotes determined (Q)? Use the symbolic environment of CAS to isolate, label, and factor the denominator.

> d:= denom(f(x));

$d := x^5 - 6x^4 + 5x^3 + 26x^2 - 48x + 18$

> factor(d);

$(x^3 - 4x + 2)(-3 + x)^2$

The double root at x=3 confirms the existence of a vertical asymptote there. The instructor draw this asymptote on the chalkboard and then focuses on the first operand. Since the CAS could not factor it, use the CAS to isolate and label it and then solve for its zeros.

> h:= op(1,")

$h := x^3 - 4x + 2$

> solve(h=0,x):

MESS!

Although the CAS gives the three zeros, the expressions contain several terms involving cube roots of imaginary numbers. (What do we do with this result (Q)?) Neither the various simplifying commands nor the evaluation command yield useable information. It is not even clear if the zeros are real, although at least one has to be real (Q). How else can the zeros be found (Q)?

Use the graphical environment of CAS to plot the graph of h. The graph, using the default domain of [-10,10] indicates that there are zero(s) in [-5,5]. Replotting over [-5,5] indicates that there are three zeros in [-2.5,2.5], but does not clearly distinguish them. Plotting again, this time over [-2.5,2.5] enables the zeros to be approximated by -2.2, 0.5, and 1.7. The instructor now draws the vertical asymptotes on the chalkboard.

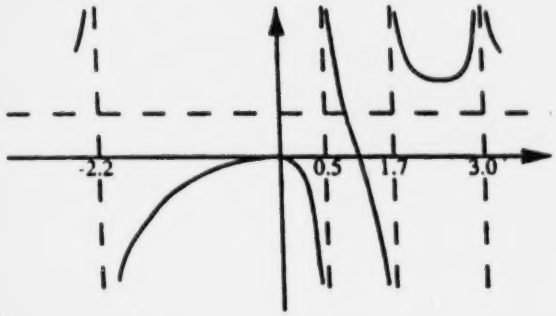
Having obtained the vertical asymptotes, what is the next step (Q)? It is clear by inspection that y = 1 is a horizontal asymptote (Q). The instructor now draws the horizontal asymptote on the axes. What is the next step (Q)? It is clear by inspection, that 0 is an x-intercept of even multiplicity (4) and 1 is an x-intercept of odd multiplicity (1) (Q). Thus the graph crosses the x axis at x=1, but does not cross at x=0 (Q).

What is the next step (Q)? Can any of the sketch be drawn? Since the multiplicities of the zeros giving the vertical asymptotes and the intercepts are known, knowledge of the numerical sign near an asymptote would allow the sketch to be drawn over the interval (-2.2–, 3+) (Q). Use the numerical environment of the CAS to evaluate f at x=3.1.

> f(3.1);

1000.151668

Thus f(3.1) is positive and so the curve rises to positive infinity as x approaches the vertical asymptote at x=3 from the right-hand side. Knowing the multiplicities, the following portion of the curve can be sketched (on the calkboard) (Q).



Note that the minimum value over [1.7, 3] is not known and thus it is not known whether the curve meets the horizontal asymptote over this interval.

What is the next step (Q)? How does the curve approach the horizontal asymptote? How do we find out (Q)? The curve meets the asymptote where f(x) = 1 or where f(x) - 1 = 0. Thus we look for the zeros of f(x)-1 = 0. These are just the zeros of

numerator(f) - denominator(f) = 0

(Q).

Use CAS to find and label this equation.

```
> n:= numer(f(x));
```

$$n: = x^5 - x^4$$

```
> g:= n - d;
```

$$g:= 5x^4 - 5x^3 - 26x^2 + 48x - 18$$

(this may require using the simplifying command). From the above sketch, we know that g has a zero in the interval [0.5,1.7]. Thus it has either one or three more zeros (Q). How can the CAS be used to find the remaining zero(s) (Q)? The CAS probably cannot factor g. However, the solve command yields a listing of four very complicated looking expressions. The numerical environment of the CAS can be used to evaluate each of these zeros to a decimal expression. The commands are:

```
> r:= solve(g=0,x);
> r1:= evalf(r[1]);
        complex number
> r2:= evalf(r[2]);
        complex conjugate of r1
> r3:= evalf(r[3]);
        .541273491
> r4:= evalf(r[4]);
        -2.608074669
```

We can now complete the sketch (Q). Note that the curve does not meet the horizontal axis over [1.7, 3].

An alternative method for finding (approximating) the zeros of g would have been to use the graphical environment of the CAS.

If time permits, this exercise can now be extended to refine the sketch by including local information (local extrema, inflection points).

[6] Subscription List and Data Bank

A <u>new</u> subscription list is being compiled for the CASE NEWSLETTER. To have your name placed on the new list, fill out and return the Subscription Form on the back page. Future mailings will <u>use only the new list</u>.

A Data Bank of CAS users (and friends) will be compiled from the new subscription list.

To receive future issues of CASE, fill out and return this form to:

Don Small
CASE NEWSLETTER
Dept. of Mathematics
Colby College
Waterville, ME 04901

Telephone: (207) 872-3255
UUCP electronic mail: ihnp4!dartvax!colby!case

Name:

Dept:

Address:

City: **State:** **ZIP:**

Telephone: **Electronic Mail:**

Check or fill in the appropriate line:

Current Use of CAS: CAS_____ Computer_____
Calculus: class___ Lab___ Lin. Alg.: class___ Lab___
Diff. Eq'ns: class__ Lab___ Discrete: class___ Lab___
Other:_____

Planned Use of CAS: Year____ CAS_____
Computer_____
Calculus: class___ Lab___ Lin. Alg.: class___ Lab___
Diff. Eq'ns: class___ Lab___ Discrete: class___ Lab___
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Suggestions for the CASE NEWSLETTER:_____

Computer Algebra Systems in Education NEWSLETTER*

Dept. of Mathematics, Colby College, Waterville, ME 04901
Number 6, June 1989

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[1] Calculus with a CAS Lab

Don Small
Colby College

The best experience that I have ever had teaching an elementary calculus course is the way I describe last year's experiment of incorporating a weekly lab into a standard calculus course. Students found the labs to be challenging, frustrating, demanding, etc., but never boring. They were involved, often times intently involved, in the mathematics in contrast to their passive role in the classroom.

Students worked in pairs at Mac II's using the Maple CAS. Although the lab was scheduled for an hour and a half, most students stayed for over two hours. Students arriving for lab were given a handout that normally consisted of a one or two-page discussion of the lab topic, exercises to be worked during lab, and homework exercises to be handed in the following Monday (see [2] and [3]). Some lab topics were chosen to introduce ideas before they were addressed in class and others were chosen to follow on class discussion.

My role in lab became one of a guide and resource person rather than an instructor. At first, giving up the "control" that I have in the classroom when I hold the chalk was a bit scary. Also, not always being able to explain what the computer had done was hard on my ego. Getting over the "control" and "ego bit" were important steps toward engaging students in "doing" mathematics as well as making the course much more fun and interesting for everyone.

Students usually began a lab "working over" the discussion part of the handout. The first fifteen minutes of a lab would restore my faith that students *can* read and learn from written material. The interaction between students working together was exciting to

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*Sponsored in part by a grant from the Sloan Foundation

watch. Students demonstrated support and interest in helping their peers. There was lots of talking and arguing (about mathematics). I saw a student actually pull her partner's hand away from the keyboard saying, "Stop, tell me what you are doing!" Active learning (by anyone's definition) was characteristic of the labs.

The types of question changed as the course progressed. By the end of the first semester only a few "How to" but several "Why" type questions were being asked, a reversal of the situation in the beginning of the semester. Use of the computer also changed during the course. In the beginning, students would often supplement the computer work with their own pencil-and-paper computations (or vice-versa). As confidence developed, this stage gradually evolved into one in which the students tried to do everything on the computer. By the end of the course, students were beginning to show signs of being more selective in their use of the computer.

I believe working on multistep problems in the multimodal environment provided by the CAS enabled students to develop their problem-solving skills better than has been the case in previous classes. Graphical analysis often became the starting point in analyzing the behavior of a function. It was exciting to watch a student discover (completely on her own), that the plot of the derivative function indicted not only the critical numbers, but also the nature of the function at each of the critical numbers.

Perfection has yet to be obtained however. Although students were more involved in doing mathematics in lab than during class, their approach was still "answer oriented" rather than "concept investigation." Furthermore students often did not seem to connect the lab work with the classwork. Next year I intend to spend more class time discussing lab activities.

As Henry Ford reluctantly learned that not everyone enjoyed black as a car color, I have to reluctantly admit that not all students demonstrated unlimited enthusiasm for the calculus lab. Feelings of frustration and insecurity often arose when students were asked to analyze situations that did not fit standard algorithms. A student wrote on an evaluation:

" I must admit that our use of computers in this class was one of the most frustrating things I have ever done. This I believe, is due to the computer's ability to give an answer that the student must figure the origins of for himself. The computer can obviously do more complicated problems, but actually makes the student learn more than he would doing problems by hand. Adding a computer lab to this course added an extra dimension to my learning of calculus that, now its over, I can appreciate greatly. Although I didn't much like this aspect of the course, I think that I have an advantage over those students who didn't have the advantage of using these machines."

Neither "computer anxiety" nor lack of prior experience (computer work in high school) seemed to be a problem. I attribute this to my being available in the lab for questions and to the first lab, designed to acquaint the student with the Maple CAS and to build student confidence in using the computer to do mathematics. The first lab is critical for establishing a positive attitude for using a computer.

Lab reports offer a nice opportunity for incorporating more student writing into our courses. Assignments that require students to explain, in writing, the reasoning for what they did and an interpretation of their results are not restricted to lab reports. However, lab reports are well suited to this type of assignments.

The following two articles give examples of labs. The first ([2]) was assigned near the middle of the course and the second ([3]) was assigned near the end of the course. This second lab closely followed a class discussion and thus there was no introductory material included in the handout. The Homework Exercise ([6]) is an example of the type of discovery problem that would be include in lab homework assignments.

[2] Lab: Analysis of an Integral

Since an integral of the form $\int_a^x f(t)\,dt$ defines a function, we may attempt to analyze it as we would any other function. That is, determine if the function is continuous, differentiable, integrable; determine where the function is monotone; determine the extreme values; determine the graphical properties --concavity, intercepts, asymptotes; etc. If we can find an antiderivative of the integrand, then we say that we can evaluate the integral in closed form. Analyzing an integral that cannot be evaluated in closed form offers a richer and more challenging experience than is the case when the integral can be evaluated in closed form. A good practice procedure is to analyze a known integral (one that can be evaluated in closed form) without first integrating and then use the integrated form as a "check".

Example: Consider the function f:[-3,5] --> R defined by $f(x) = \int_2^x (t^2+2)\,dt$. Sketch the graph of f without evaluating the integral. Then use the plot of the evaluated form of the integral to check your sketch.

By inspection, we see that:

(i) since the integrand is continuous, f is differentiable and thus continuous,
(ii) f(2) = 0, and
(iii) f is negative for x < 2 (since the integrand is positive).

Applying the Fundamental Theorem of Calculus yields, f'(x) = x^2+2 > 0. Thus f is monotone increasing over the interval [-3,5]. Hence f has a minimum value at x=-3 and a maximum value at x=5. Furthermore the slope of the tangent line to the graph is 11 when x=-3, 2 when x=0, and 27 when x=5.

Using Simpson's Rule with 50 subdivisions, we evaluate f(-3) and f(5).

```
> -simpson(x^2+2, x=-3..2, 50);      -21.6667
> simpson(x^2+2, x=2..5, 50);         45.0000.
```

(Why is there a negative sign before the first simpson command, but not the second?) We determine concavity by looking at the second derivative (the first derivative of the integrand):

```
> diff(x^2+2, x);          2x.
```

Thus the graph has a negative concavity over [-3,0), positive concavity over (0,5], and a point of inflection at (0,0). To "tie down" our sketch, we evaluate f (via Simpson's Rule) and the slope of the tangent line at a few points, say x=-1 and 3.

We now sketch the graph and "check" it with a CAS plot.

```
> plot(int(t^2+2, t=2..x, x=-3..5));
```
(sketch and plot omitted)

Exercise (to be done during Lab)
Make up an interesting (i.e., non-trivial) example of a function defined by an integral that can be evaluated in closed form. Sketch the graph of your function without evaluating the integral and then check your sketch with a CAS plot of your function. Compute all numerical approximations to your function using an accuracy of 0.1. When you have finished "checking" your sketch, list three ways in which you could have made your function "more" interesting (explain why the modified functions are more interesting).

Homework Exercise
Give as complete an analysis as you can of f:[1,20]-->R defined by $f(x) = \int_1^{x^2} \sqrt{t}\sin(t)\,dt$. Your analysis should include (but not be limited to) the graph of the function. Describe (in written form) what you are doing, why you are doing it, and your results at each stage of the analysis. You should expect to rewrite your description several times before obtaining a clear and understandable copy (do not hand in a "draft.").

[3] Lab: Playing with Harmonic Series

The purpose of exercises #1-5 is to develop a "feeling" for the behavior of the *harmonic series*, $\sum_{k=1}^{infinity} 1/k$.

1. Is (s_n) a decreasing sequence? Why?

2. Compute s_n for n = 10, 50, 100, 10^5, 10^{10}, 10^{100}, $10^{1,000}$, $10^{10,000}$. What is the effect on s_n when n increases by a power of 10? Explain.

3. How does the rate of growth of (s_n) compare with the rate of growth of log(n)? Explain.

4. Show that $\log(n) + 1/n < s_n < 1 + \log(n)$. (Think of log(n) as the area "under the curve" of f(x) = 1/x over the interval [1,n]. Draw pictures to illustrate upper and lower approximating sums.)

5. Using log(n) to approximate s_n, determine a value of n such that $s_n > 100$. Does using log(n) help explain the pattern you observed in #2? Explain.

The purpose of exercises #6-10 is to develop a "feeling" for the *alternating harmonic series* $\sum_{k=1}^{infinity}(-1)^{k+1}1/k$.

6. Compute s_n for n = 2, 4, 6, 8, 10. Is (s_{2n}) an increasing sequence? Explain.

7. Compute s_n for n = 1, 3, 5, 7, 9. Is (s_{2n+1}) a decreasing sequence? Explain.

8. What relationship (if any) exists between (s_{2n}) and (s_{2n+1})? Explain.

9. Assume $\sum_{k=1}^{infinity}(-1)^{k+1}1/k$ converges and has sum L. What relationship exists between (s_{2n}), (s_{2n+1}), and L? Explain. Use this relationship to obtain an error bound for $|\sum_{k=1}^{infinity}(-1)^{k+1}1/k - s_{20}|$. Explain.

10. Show that the alternating harmonic series converges to log(2). (Hint: expand log(x) in a Taylor series about x=1.) Determine n such that s_n for the alternating harmonic series approximates log(2) with accuracy = 0.01.

[4] Thrust of the 1989 Round of Calculus Proposals to the NSF

J.S. (Spud) Bradley
National Science Foundation

The calculus reform movement is much broader than the program sponsored by the National Science Foundation; the reform movement began before the NSF curriculum development program began and much outstanding progress is being made without NSF support. Nevertheless, the proposals submitted to the Foundation represent a broad cross section of efforts. Additionally, the projects funded by the NSF are excellent, good progress is being made, and the NSF program is contributing to the momentum of reform.

The purpose of this article is to attempt to present a broad brush picture of the proposals that were submitted in 1989. Any such summary will miss some important ideas and will oversimplify others. The aim is to capture the primary thrusts.

* Projects are based on experience, not conjecture. Very few proposals could be characterized by "I have a good idea, give me some money to try it". Most show evidence that the proposer has given the project considerable thought and has tested the ideas in the classroom. Often plans have been changed because what appeared at first to be a good idea did not work.

* Calculus is seen as a laboratory course. Many projects are built around the idea that students should formulate many of the basic concepts themselves, or at least confirm these concepts, based on experimentation much as in a chemistry course. For example, graphing experiments using computers or graphing calculators can lead to meaningful conjectures about the form of Taylor's Theorem. By looking at the graph of sin(x) near the origin it is easy to guess the linear approximation. Graphing the difference of sin(x) and the linear approximation suggests the form of a cubic. Students easily recognize these graphs from previous experiments.

* Use of computers and calculators is increasing. These tools are becoming more and more widely used. The laboratories mentioned earlier are computer or calculator laboratories. These devices allow students to display a graph quickly and almost painlessly so many graphs can be used to explore properties of functions thoroughly.

* There is more student involvement and less lecturing. A ten-minute lecture is considered a long lecture in many of the projects. Students are getting more involved during the class sessions and the lab sessions and are no longer passive listeners from whom the lecturer may occasionally try to elicit a question. The higher level of involvement is compatible with the laboratory setting and with the use of computers and calculators. An observed consequence of this is more excitement about calculus on the part of the students.

* Expectations have been elevated. Those directing the projects are now expecting a

higher level of performance from the students and are getting it. More difficult problems are assigned, especially where several days or weeks are allowed and/or students are allowed to work in teams. Students are usually expected to present their solutions in well-written English sentences. Project directors are finding that well planned uses of computers and/or calculators contribute significantly to this process of expecting and getting better performance from the students.

* The content of calculus is leaner, but recognizable. Most of the new projects have dropped some topics from the traditional calculus curriculum and treat other topics in non-traditional ways. However, the change in topics is not dramatic.

(Spud Bradley is the Program Director for Mathematics, Division of Undergraduate Science, Engineering, and Mathematics Education, the National Science Foundation.)

[5a] Newsletters

Computer Algebra Systems in Education Newsletter
(which you are reading)

The Maple Newsletter
Brooks/Cole Publishing Company
511 Forest Lodge Road
Pacific Grove, CA 93950
(408) 373-0728

duke/ncssm calculus newsletter
Project CALC
Department of Mathematics
Duke University
Durham, NC 27706

Lagrange Calculus
Mathematics Department
Community College of Philadelphia
1700 Spring Garden St.
Philadelphia, PA 19130
(215) 751-8413

[5b] CASs

Derive, muMath:
Soft Warehouse
3615 Harding Ave., Suite 505
Honolulu, Hawaii 96816
(808) 734-5801

Macsyma:
Symbolics, Inc.
Eleven Cambridge Center
Cambridge, MA 02142
(617) 621-7500

Maple:
Watcom Products, Inc.
415 Phillip Street
Waterloo. Ontario
Canada N2L3X2
(519) 886-3700

Maple (MacIntosh)
Brooks/Cole Publishing Co.
511 Forest Lodge Road
Pacific Grove, CA 93950
(408) 373-0728

Mathematica:
Wolfram Research, Inc.
P.O. Box 6059
Champaign, IL 61820
(217) 398-0747

Reduce:
Northwest Computer
Algorithms
P.O. Box 1747
Novato, CA 94948
(415) 897-1302

SMP:
innosoft international inc.
250 W. First Street, Suite 240
Claremont, CA 91711
(714) 624-7907

[5c] Workshops

The following CAS Workshops are supported by an National Science Foundation grant. (All expenses, except for participant travel, are covered by the grant.)

July 23-28, 1989: (5-day)
St. Olaf College, Northfield, MN 55057
Instructors: Paul Zorn, Mike Henly
Contact Person: Arnie Ostebee

Colby College, Waterville, ME 04901
Instructors: Don Small, Doug Child

July 14-15, 1989:
Univ. of Portland, Portland, OR
Instructor: Doug Child
Contact Person: Bruce Char

Oct. 13-14, 1989:
Mississippi State, MS 39762
Instructor: Wade Ellis, Jr.
Contact Person: Jimmy Soloman
(601 - 325-3414)

Nov. 3-4, 1989:
Colby College, Waterville, ME 04901
Instructor: Don Small
Contact Person: Bruce Frech
(207 - 872-3259)

Nov. 10-12, 1989:
New York City Technical College
Instructor: Warren Page
Contact Person: Geoff Akst
(212 - 618-1862)

There will be at least three 5-Day Workshops presented during the summer of 1990.

[6] Homework Exercise (discovery type).

(1) Let $B_n = \int_0^\infty x^n e^{-x}\,dx$.

a. Use a CAS to compute B_n for several successive values of n.

b. Conjecture a closed form expression for B_n based on your results in part a.

c. Test your conjecture by using your CAS to compute B_n for a value of n different from those selected in part a.

d. Prove that your conjecture is correct. (Hint: try using Integration by Parts.)

(2) Replace "x" by "ax" in the expression for B_n and repeat steps a - d.

(3) Replace "n" by "bn" in the expression for B_n in (1) and repeat steps a - d.

(4) Replace "n" by "bn" in the expression for B_n in (2) and repeat steps a - d.

[7] To receive future issues of CASE, complete and return this form to:

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Dept. of Mathematics
Colby College
Waterville, ME 04901

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Name:

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Address:

City: State: ZIP:

Telephone: Electronic Mail:

Check or fill in the appropriate line:

Current Use of CAS: CAS_____ Computer_____
Calculus: class___ Lab___ Lin. Alg.: class___ Lab___
Diff. Eq'ns: class___ Lab___ Discrete: class___ Lab___
Other:_____

Planned Use of CAS: Year____ CAS_____ Computer_____
Calculus: class___ Lab___ Lin. Alg.: class___ Lab___
Diff. Eq'ns: class___ Lab___ Discrete: class___ Lab___
Other:_____

Interest in contributing to the CASE NEWSLETTER: Yes ___ No ___
Uncertain___

Suggestions for the CASE NEWSLETTER:_____

Computer Algebra Systems in Education NEWSLETTER*

Dept. of Mathematics, Colby College, Waterville, ME 04901
Number 7, January 1990

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[2] The Implicit Function via CAS

[3] An Interesting CAS Project

[4] A CAS Lab Exercise

[5] Requests and CAS Workshops

[6] Subscription Form

[1] Analyzer* - An Inexpensive, Easy-to-Use CAS for the Macintosh

Richard E. Sours
Wilkes College

Analyzer* is billed by its authors as "A Macintosh Program for Calculus Students." It is exactly that. It is not a comprehensive, general purpose mathematics program. It is designed to do Calculus, and it achieves that goal very well. Its low cost and ease-of-use make Analyzer* very attractive for the first-year course.

Advantages and Capabilities

Analyzer* costs $35 and requires a 1MB Macintosh. The accompanying documentation is only 60 pages, with lots of pictures (of screens and menus). Students who had never used a Macintosh felt "comfortable" with Analyzer* after 2-3 hours, and considered themselves "pros" after 4-5 hours. There is one basic screen which is used for data entry, graphical output, and as a source of menus for doing a variety of numeric and symbolic tasks. There are no commands to learn and, except for entering functions, no typing is required. All tasks are done in response to "mouse clicks."

Analyzer* will plot (in two dimensions) the graphs of functions (of one variable) using any domain and range. It will overlay the graphs of several different functions on one screen, and it will plot a function and its successive derivatives on the same screen. There is a "Blow-Up" feature which (by dragging the mouse) allows the user to zoom in and enlarge one portion of the graph to fill the entire screen. When in the graphing window, the cursor is a pointer arrow. A mouse click gives the x and y coordinates of the arrow head. This is ideal for such tasks as estimating $\lim_{x \to a} f(x)$.

Analyzer* does symbolic differentiation, finds roots using either the bisection methods or Newton's method (user's choice), and finds extrema using the bisection method on the first derivative. All of these

*Sponsored in part by a grant from the Alfred P. Sloan Foundation

may be displayed graphically or numerically. The program will do numerical (but not symbolic) integration using any of five methods (Righthand, Lefthand, Midpoint, Trapezoid, or
Simpson's) with any number of "panels" (i.e., subdivisions of the interval of integration). Virtually all of the functions used in calculus are built in (even greatest integer and $\sinh^{-1}(x)$). Finally, the user can adjust the number of decimal places used in computation and/or display, change the scale on either axis from decimal to base p or base e, and print any screen in a variety of colors (given a suitable printer).

Limitations and Disadvantages

Symbolic integration would be nice, as would 3-D graphics, but these features would raise the cost in both dollars and system requirements. There are no user-specified capabilities such as algebraic simplification (i.e., no user programming). Version 2.0 has removed some of the quirks and added some capabilities not available in earlier versions. In particular, it is now very easy to specify the endpoints of integration. Also, it is easier to enter "multi-barrelled" functions such as:

$$f(x) = \begin{cases} 3 & \text{if } x \le -2 \\ 4 - x^2 & \text{if } -2 < x < 1 \\ 2x + 1 & \text{if } x \ge 1 \end{cases}$$

On balance, Analyzer*'s many advantages far, far outweigh its few shortcomings, and it is a very good choice for Calculus. The short learning curve and ease of use mean that students can concentrate on Calculus without being distracted by syntax errors or by having to remember specific commands.

For further information, contact:

Beverly West or Andrew Kaplan
Dept. of Mathematics
Cornell University
Ithaca, NY 14853
(607) 255-7244

[2] The Implicit Function via CAS

Robert J. Lopez
Rose-Hulman Institute of Technology

The challenge for instructional use of computer algebra is not technical, but pedagogical. Teaching the syntax of differentiation and integration is a rote task. Devising effective activities that lead to increased conceptual understanding is creative.

(1) $3x^2 + xy^2 - 8y = 0$

(2) $8x^2 + 12xy + 17y^2 - 28x - 46y + 17 = 0$

lead to

(3), $y(x) = [4 \pm \sqrt{16 - 3x^3}]/x$

(4) $y(x) = [23 - 6x \pm 2\sqrt{5}\sqrt{10x + 125x^2}]/17$

respectively. Note how the concept of branches arises naturally in these computations.

Equally important, the identity $f(x, y(x)) = 0$ is now explored by substituting $y = y(x)$ into $f(x, y)$ and simplifying.

Always useful and often impressive, a graph of $y(x)$ is easily obtained when $y(x)$ is known explicitly. Thus, (4) represents the upper and lower halves of a rotated, translated ellipse.

Finally, it is instructive to consider examples such as

(5) $\sqrt{3x^2 - 3xy - 2y^2}e^{9/\sqrt{33}\operatorname{arctanh}(3x+44/\sqrt{33}x)} - 187 = 0$

where $y(x)$ cannot be obtained explicitly. Consistent with the meaning of the implicit function, an x-value is selected and substituted into (5) and the resulting equation $g(y) = 0$ is numerically solved for y. The built-in numeric solver or a user-created procedure, such as Newton's method, can be used. Thus, a table of values is constructed and a graph obtained.

In Maple, the first ten Newton iterates, starting from $y = a$, are produced by the commands

$q := y - g/diff(g, y);$

a;
for k from 1 to 10 do evalf(subs($y =''$, q)); od;

The Maple command
plot($[x1, y1, x2, y2, ..., xn, yn]$)
generates a (spline) plot of the discrete data points $(x1, y1), ..., (xn, yn)$.

Moreover, the CAS Mathematica has a contour plot command that would graph y(x) as a contour of the surface $z = f(x, y)$. The curricular implications are now significant. But whether or not a discussion of surfaces is moved from its traditional place in Calculus III to Calculus I, the pedagogical challenges inherent in the use of computer algebra are clearly substantial.

[3] An Interesting CAS Project

(Some New Math ?)
Bill Stearns
University of Maine

At a CAS Workshop I was told that if one gave students a CAS system the students could and would develop some new mathematics by experimentation. I was my normal cynical self until it happened to me.

While preparing some computer graphics for a class discussing L'Hospital's Rule, I discovered the following behavior. So far I have found no one who recognizes it as an item from his/her mathematical background, hence one may be able to claim it as a new mathematical curiosity.

Compare any two functions $n(x)$ and $d(x)$ such that $n(x)/d(x)$ satisfies the conditions of L'Hospital's Rule for the 0/0 case as x approaches a finite number, preferably the conditions for repeated application of the Rule.
Compare the graphs of n/d, n'/d', n''/d'', etc. about $x = a$ and notice the new aggressiveness with which the successive ratios approach the limit.

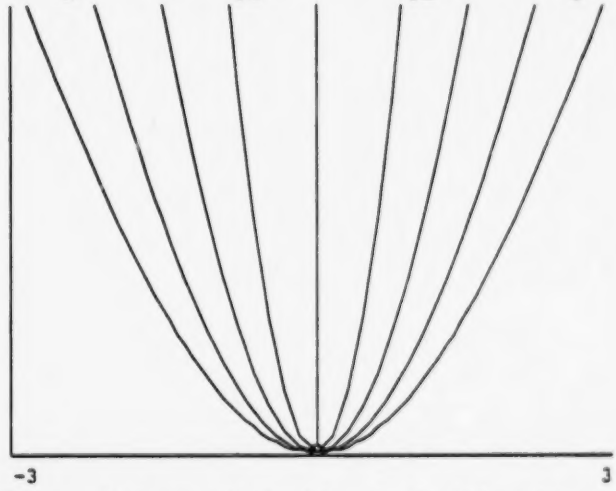
For example,
try $n(x) = \sin(x) - x$, $d(x) = x^3$
and $n(x) = 2x^3 + x^4$, $d(x) = x^3 + 2x^4$.

Be sure your domain is large enough to see the shapes of the curves. CALC-PAD does the set up very nicely.

The use of infinite series representation for $n(x)$ and $d(x)$ and then for $n(x)/d(x)$ and $n'(x)/d'(x)$ will easily verify the result for some cases. In general, it is more troublesome.

Quiz: In the following figure, identify the graphs of

$$\frac{\sin(x) - x}{x^3}, \quad \frac{\cos(x) - 1}{3x^2}, \quad \frac{-\sin(x)}{6x}, \quad \frac{-\cos(x)}{6}.$$



Enjoy playing with the idea, it should make a nice CAS lab assignment. I would be interested in seeing any lab project work-ups and /or student results that develop from this. (Bill Stearns, Mathematics Department, Univ. of Maine, Orono, ME 04469)

[4] A CAS Lab Exercise

Don Small
Colby College

The numerical signs of the first and second derivative of a function provide useful information about

the behavior of a function for finding extremal values as well as for predictive purposes. Since the the second derivative predicts something about the behavior of the first derivative and thus the function in the "near" future, the second derivative of a function of time is often interpreted as indicating a future trend. Consider for example, the sine function $f(x) = \sin(x)$. Does the fact that $f''(x) = -f(x)$ help explain why the sine function is periodic?

The purpose of these exercises is to develop (or confirm) mental pictures of the shapes of graphs based on the numerical sign and magnitude of the first or second derivative.

1. For each of the following sets of conditions, define three functions that satisfy the conditions (remember to specify the domains), draw and label a multiplot of their graphs, and then conjecture (in writing) a description of the shape of the graph of a function that satisfies the stated conditions. If no function satisfying the conditions can exist, explain

a. Functions that have a positive first derivative.

b. Functions that have a negative first derivative.

c. Functions that have a positive second derivative.

d. Functions that have a negative second derivative.

e. Functions that have a positive first derivative and a positive second derivative.

f. Functions that have a negative first derivative and a negative second derivative.

g. Functions that have a positive first derivative and a negative second derivative.

h. Functions that have a negative first derivative and a positive second derivative.

2. Follow the instructions for Exercise 1.

a. Functions that have a positive and bounded first and a positive second derivative.

b Functions that have a positive and bounded first and second derivative.

c. Functions that have a positive first derivative, negative second derivative, bounded first derivative, and unbounded second derivative.

d. Functions that have a negative first derivative, positive second derivative, unbounded first derivative, and bounded second derivative.

3. (Open Ended) If f is a differentiable function defined over (a, b) and $|f'(x)| < 10$, what can you say about the range of f? Describe your experiments and conjectures.

4. (Open Ended) Let f and g be differentiable functions defined over (a, b) and let $f'(x) < g'(x)$ for all x in (a, b). Is it true that $f(x) - f(a) < g(x) - g(a)$ for all x in (a, b) ? Why?
Hint: Is the function h defined by $h(x) = g(x) - f(x)$ an increasing function?

[5] Requests

Ben F. Zirkle is interested in making contact with anyone using or planning to use a CAS in a mathematics program for two-year engineering technology majors. (Mr. Ben F. Zirkle, Mathematics and Science Division, Virginia Western Community College, P.O. Box 14007, Rianoke, VA 24038)

Articles, problems, labs, etc. for inclusion in future issues of the CASE Newsletter are always welcome. Send them to Don Small, Colby College, Waterville, ME 04901

CAS Workshops

July 15-20, 1990 Colby College, Waterville, Me. 04901 Contact: Don Small

July 22-27, 1990 St. Olaf College, Northfield, MN 55057 Contact: Arnie Ostebee

[6] To receive future issues of CASE, complete and return this form to:

Don Small
CASE NEWSLETTER
Dept. of Mathematics
Colby College
Waterville, ME 04901

Telephone: (207) 872-3255
UUCP electronic mail: dbsmall @colby.UUCP

Name:

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Address:

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Check or fill in the appropriate line:

Current Use of CAS: CAS_____ Computer_____
Calculus: class___ Lab___ Lin. Alg.: class___ Lab___
Diff. Eq'ns: class__ Lab___ Discrete: class___ Lab___
Other:_____

Planned Use of CAS: Year____ CAS_____
Computer_____
Calculus: class___ Lab___ Lin. Alg.: class___ Lab___
Diff. Eq'ns: class___ Lab___ Discrete: class___ Lab___
Other:_____

Interest in contributing to the CASE NEWSLETTER: Yes ___ No ___
Uncertain___

Suggestions for the CASE NEWSLETTER:_____